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MODELING THE EUTROPHICATION PROCESS

Florida University
Gainesville, Florida

1969

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MODELING THE EUTROPHICATION WORKSHOP

ERRATA SHEET

Page Number	Paragraph	Should Read
81	4	according to trophic levels
87	1	numerous variables
92	2	domestic sewage
94	3	origin on Figure 4
115	2	Empirical models
115	3	as deterministic
115	3	population dynamics
116	2	Fish camp owners
117	1	on occasion
124	2	heterotrophic organisms
127	1	influenced greatly
139	3	depicted as an "analytical balance"
144	2	which significantly affect
145	2	major sources
155	3	available carbonaceous
159	2	steady state condition
162	Table 3	** Parameter is varied over indicated range.
166	1	homogeneous environment
177	3	indices for the most part
179	2	individual indices
184	2	oligotrophic-high color
201	Table 1	terrestrial
203	Table II	aquatic vegetation
204	Table III	diversity indices
204	Table IV	Optimum number of groups.
208	Table VI	total variance
214	1	broad summary statements
215	2	multivariate techniques
215	2	canonical analysis
222	4	multivariate techniques
231	1	Monod (1949)
232	1	(Malek and Fencl, 1966)
238	2	Porcella (see Middlebrooks, 1969)
239	1	Porcella (see Middlebrooks, 1969)
271	1	land values
283	2	control nitrogen and phosphorus
284	4	chart shown on page 273.
287	1	only one substantial
287	1	phosphorus input

II B

PB 217 383

MODELING THE
EUTROPHICATION PROCESS

Proceedings of a Workshop
at St. Petersburg
November 19-21, 1969

This project was sponsored by
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Federal Water Quality Administration,
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College of Engineering
University of Florida

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this document may be better
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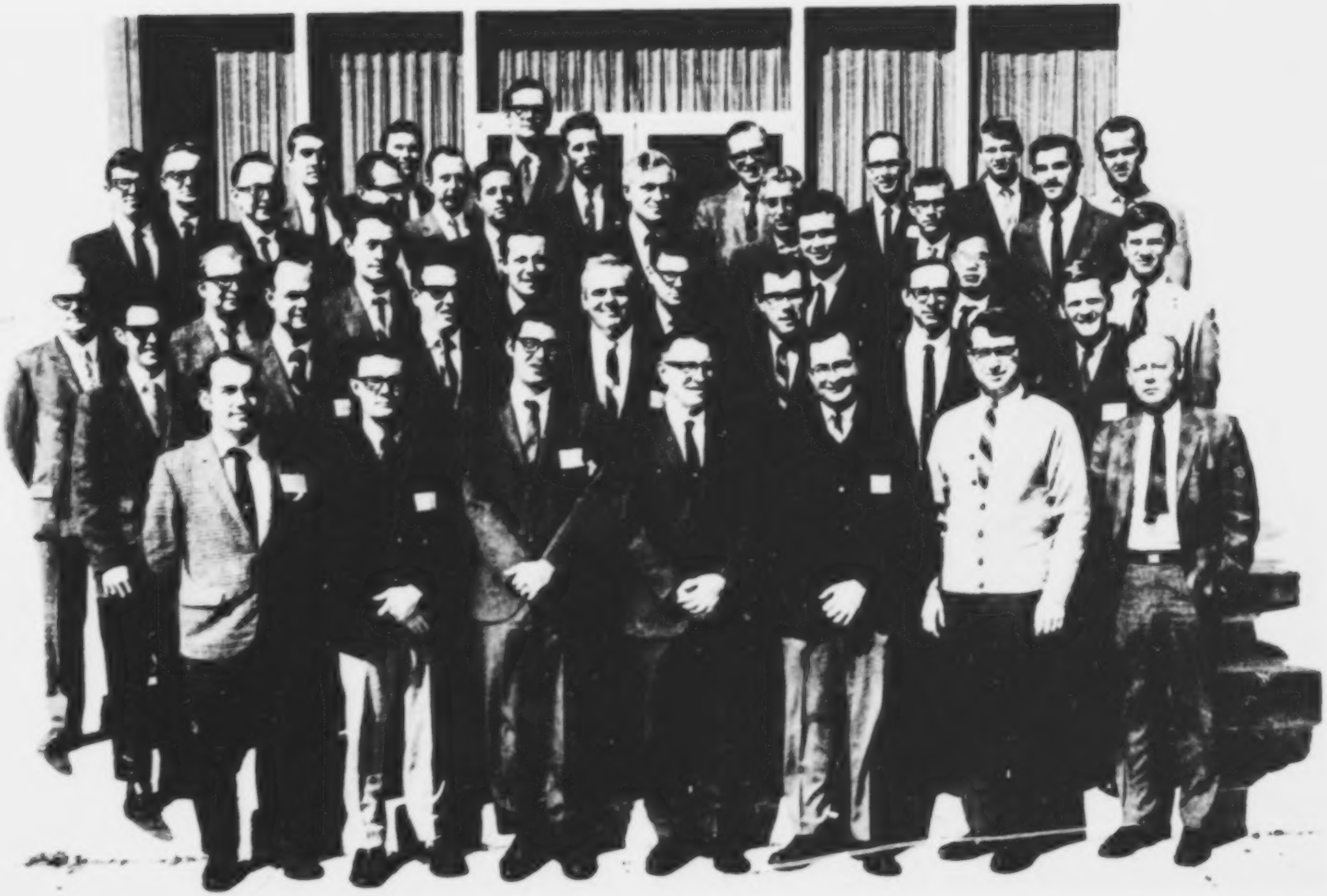
I

WORKSHOP PARTICIPANTS

St. Petersburg, Fla. - November 1969



First Row, Left to Right

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FOREWORD

In 1968, at Madison, Wisconsin, a small group attending the annual ASLO conference met informally to discuss an information exchange mechanism for those investigators working in the area of lake eutrophication. Dr. Ken Malueg representing the National Eutrophication Laboratory, FWQA conveyed the ideas of the group to Dr. A. F. Bartsch and others in his agency.

Out of this initial meeting the concept of eutrophication workshops was formalized in July 1969 and the first of these on Biostimulation was jointly sponsored by the University of California and the National Eutrophication Research Program in Berkeley.

The second conference on modeling was planned during the summer of 1969 and formally structured in the fall of that year. The number of attendees for this workshop was purposefully kept small and organized into panels so that total participation would be achieved. This approach, provided an atmosphere for free discussion which is included, in part, in these proceedings. In addition, at the conclusion of the session considerable interest was voiced for further workshops in what could be termed systems limnology. Hopefully, an additional working session will convene sometime in the near future where the modeling concepts brought forth in this workshop can be applied to existing eutrophication problems.

As with all meetings a great deal of work is done by many people who usually are unrecognized for their help in arranging for facilities as well as completing the many small tasks that are necessary for a successful conference. Special acknowledgment is made to Dr. Harold Humm, University of South Florida, Dr. W. H. Morgan, Mrs. Jeanne Dorsey, Mrs. Carol Moore, Mrs. Wm. Grantham, University of Florida, and Mrs. M. Y. Foger, St. Petersburg, Florida, for the many hours spent in preparation of the workshop. Without their help the success of the meeting could not have been achieved.

H. D. PUTNAM
Chairman

EUTROPHICATION WORKSHOP SOUTHEAST
ST. PETERSBURG, FLORIDA
NOVEMBER 19 - 21, 1969

MODELING THE EUTROPHICATION PROCESS

Tuesday, November 18
6:00 - 8:00 p.m. - Registration at Edgewater Beach Motel

Wednesday, November 19
8:00 a.m. - Welcome

H. D. Putnam
A. F. Bartsch

Session I - Concepts of Mathematical Models

D. H. Moreau - panel leader
J. F. Burns
C. Chen
W. S. Galler
D. A. Bella

Session II - Eutrophication: The Process and its Modeling Potential

P. L. Brezonik - panel leader
J. J. Morgan
E. G. Fruh
G. F. Lee

Recess for Lunch

Session III - The Process of Eutrophication and Criteria for Trophic State Determination

J. R. Vallentyne - panel leader
J. Shapiro
R. Vollenweider
A. M. Beeton

Session IV - Multivariate Techniques for Classification of Lakes and the Study of Eutrophication

E. E. Shannon - panel leader
D. Watts
A. L. Sheldon
D. Robb

Adjourn

Thursday, November 20

8:00 a.m. - Reconvene

Session I - Model of Aquatic Ecosystems

B. C. Patten - panel leader
R. B. Williams
J. C. Shaake
R. A. Parker

Session II - Biogeochemical Modeling of Eutrophic Lakes

W. F. Echelberger - panel leader
J. A. Little
N. L. Clesceri
W. M. Sanders, III

Recess for Lunch

Session III - Models of Algal Population Dynamics Based on the Chemostat Approach

D. J. O'Conner - panel leader
E. J. Middlebrooks
C. M. Weiss

Session IV - Eutrophication Management Modeling

J. P. Heaney - panel leader
B. Dysart III
K. C. Gibbs
J. E. Reynolds
J. L. Fox

Adjourn

Thursday Evening - Meeting of Panels to Prepare Summary Statements

Friday, November 21

8:00 a.m. - Summary Session

Tour Local Points of Interest
a) Continuous sampling techniques for marine benthic invertebrates. Field trip to St. Petersburg Bay.
b) Busch Gardens, Tampa

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CONCEPTS OF MATHEMATICAL MODELS

A Position Paper

by

David H. Moreau

with a note

by

David A. Bella

Panel Members:

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William S. Galler, Department of Civil Engineering, North Carolina State University, Raleigh, North Carolina

CONCEPTS OF MATHEMATICAL MODELS

The real world, with its milieu of dynamic physical and social processes acting simultaneously in space, is indeed a complex entity. Few would even attempt to understand it in its totality except on philosophical or theological bases, but the physical and social sciences do have as their objectives: (1) to assess the state of the real world at any given point or points in time, and to describe how certain variables may be observed to vary in a more or less consistent relationship over time and space, (2) to unveil the causal mechanisms among those variables that tend to vary together consistently, and (3) to exploit existing knowledge and resources to serve a variety of ends. In either of these pursuits, many scientists have found the language and deductive reasoning of mathematics to be of considerable assistance in communicating their findings and drawing conclusions about the processes to which their attention is focused. Summaries of data and statements of hypotheses of real world phenomena may take the form of mathematical models that may be subsequently manipulated according to logical rules of mathematics to reach conclusions that may not be derived easily or at all from observations alone.

Mathematical models are, of course, not the only kind of model that has been of assistance in scientific endeavors. Ackoff (1962) suggests that in science, as well as in other pursuits, three general types of models are employed. They are: (1) iconic models, (2) analogue models, and (3) symbolic models. Iconic models are those where the model is a scaled replica of the states, objects or events they represent. Included among such models are road maps, aerial photographs, blueprints, flow charts, etc. Analogue models are those in which some property of the model represents a different property in the real world, e.g., voltages may be used to represent energy differences in a network transmitting fluids. Currents may be used on a map to represent elevation. Symbolic models are those in which symbols are used to represent properties of objects being represented. The special symbolic model that is examined in further detail in this paper is that one in which the symbols are used to represent variable properties of certain objects or events and how those quantities are related to the properties of other objects and events. These are the so-called mathematical models.

Handmaiden of the Sciences

Mathematics is commonly regarded as a science within the same definitions that classify physics, chemistry, biology, economics, sociology, and other fields of study within the sciences. Its origins, dating to some 3000 years ago, was the science of numbers. It, like the other sciences, is advanced through the processes of invention and discovery. Poincare (Taton, 1962) writes of mathematical discovery:

> What, in fact, is mathematical discovery? It does not consist in making new combinations with mathematical entities already known. That can be done by anyone and the combinations that could be formed would be infinite in number and the greater number of them would be absolutely devoid of interest. Discovery consists precisely in not constructing useless combinations, but in constructing those that are useful, which are an infinitely small minority. Discovery is discernment, selection...those mathematical facts are worth studying which reveal unsuspected relationships between other facts long since known...but borrowed from widely separated domains.

The question is generally raised as to the usefulness of mathematical creativity. Taton points to several examples in the history of science where discoveries in mathematics led to very fruitful applications. Again, quoting from Taton (1962).

> Conversely, many theoretical advances owe their origin to practical sources. Physics has frequently been a great help to mathematics, by suggesting the kind of problem that theoreticians would never have dreamt of, and by suggesting, out of a tremendous number of unresolved questions, those that were to prove of exceptional fruitfulness...Yet there are some who, like Newton, Gauss, Hamilton, Poincare and Cartan, have a breadth of vision, great powers of conception and considerable virtuosity, so that they can profit from the fruitful sources of inspiration: feelings for mathematical harmony and for physical reality.

Indeed, Newton found it necessary to invent new mathematics for the study of motion. As stated by Bell (1937).

> Common experience teaches us that continual, varying change is the order of nature....Until mathematics can take a grip on the elusiveness of change it is powerless to grasp nature in its most characteristic activities...motion is less mysterious...since Newton and Liebnitz developed the calculus. This penetrating instrument of mathematical thought, particularly this differential calculus, is the national language of motion and indeed of all continuous change...Over ninety-nine per cent of all the important 'laws' in physical science are subsumed in differential equations, or in systems of such.

Thus, he concludes, that while mathematics is neither a necessary nor a sufficient endeavor and deserves to be regarded as the "Handmaiden of the Sciences." The essence of mathematics is deductive reasoning from explicitly stated assumptions. The utility of mathematics is dependent first upon the genius of a researcher to describe observed dependents of variables in appropriate mathematical terms, and secondly, upon his ability to effectuate the deductive reasoning process. Fortunately, for a multitude of possible descriptions for relationships among variables, the latter process has been carefully catalogued in the mathematical literature. Furthermore, there is an accumulated experience in the development of appropriate mathematical descriptions for physical and social phenomena.

THE SYSTEM AND ITS ENVIRONMENT

In confronting the task of analysis for either pure or applied scientific purposes, analysts have found it convenient to isolate the phenomena or decision under study from other unrelated or interfering processes. The study of mechanics has utilized for years the notion of "free body" diagrams in which external forces on a structure are specified and the resultant internal distribution of forces and stresses on structural members are calculated. Laboratory studies, in which certain factors that affect the process under study are controlled, are essential to scientific discovery. These notions are closely akin to the dichotomy drawn by systems analysts in dividing the real world into two parts; first, the system, consisting of those objects and their attributes and interrelationships, that are of immediate interest, and second, the environment for that system which consists of all objects outside the system, a change in which may affect the system and which may be changed by the behavior of the system (1962). While these concepts may be intuitively inviting, because it enables the analyst to focus attention of only a part of the universe at any given time, the task of assigning an interfering object or process to either the system or its environment may be a crucial decision indeed. For if we attempt to model a system for which important interfering processes have been assigned to the environment of that system, the resulting analyses may be rendered useless. Nonetheless, the concept of modeling a system while treating its environment as known is a very useful abstraction in the analytical process.

Mathematical models may then be defined as an equation or a system of equations and inequalities that relate a collection of variables representing time, space, and attributes of objects (including their number) that constitute a system and its environment. A complete model must not only represent how variables within the system are related, but also how variables within the system are affected by environmental variables and vice versa--variables that are assigned to the environment are generally referred to as being exogenously determined while variables within the system are called endogenous variables and may, under proper conditions, be determined by the solution of the system of equations and inequalities.

The following simple example of water quality control may serve to illustrate these and other concepts to be discussed later. A waste flow containing an unstable substance is discharged from an industry to a storage basin where decay, the rate of which is responsive to temperature, is allowed to progress, if possible, to an extent that effluent from the basin can be safely disposed of in a nearby stream. The problem is to determine the time dependent behavior of releases from the storage basin so that adequate facilities may be provided. Obviously, changes could be made in the manufacturing process and characteristics of the stream flow could be altered to control the amount of waste and the quantity of water available for dilution, but for purposes of this discussion, those variables will be assigned to the environment in which this simple storage system operates. Ambient and waste temperatures will similarly be treated as exogenous variables. Analysis must then proceed to a mathematical description of how endogenous and exogenous variables are interrelated.

MATHEMATICAL DESCRIPTION

Mathematical descriptions of systems and their environments, like those of verbal descriptions, are the products of the perception and imagination of an individual or group who constructs the description. Freedom of description inevitably leads to competing mathematical models for the same or similar systems, the choice among which is dependent on the perspective of those to whom such models are useful. Consequently, there appear to be few guidelines, other than general rules of logical consistency and tractability, to direct those who construct mathematical models. The ultimate criterion for choosing among competing models may be that of how well the model predicts future behavior of the system, but that criterion is not sufficient to dictate choice because specifications on accuracy and precision for prediction vary from problem to problem. These considerations point to the futility of deriving rigorous guidelines for constructing and evaluating alternative models, but, as an alternative one can point to some concepts and principles that have been utilized frequently.

Although this listing is not intended to be exhaustive, four different types of relationships among variables are frequently found in mathematical models of systems. They are (1) mathematical statements of universally applicable (in some cases, almost universal) principles such as conservation of mass, conservation of energy, the laws of thermodynamics, etc.; (2) mathematical statements of particular relationships derived by the sciences either on the basis of deduction from universal principles or upon reproducible empirical results; (3) logically derived relationships; and (4) approximate models ranging from empirically derived relationships that have excellent predictive characteristics, despite the fact that they cannot be derived from basic principles, to relationships, such as some time series, econometric, and other statistical models, that may represent elaborate summaries of data but reveal little about causal mechanisms.

Although the first three types of relationships may appear in an endless variety of forms in specific mathematical models, further elaboration is given only by example. In the simple illustration of quality control, the principle of conservation of mass can be applied to get

Rate of change of substance in storage
= input rate - release rate - decay rate (1)

and

Rate of change of volume of water in storage
= input rate - release rate (2)

If q_I is the rate of waste flow containing the substance in concentration C, q_R is the release flow, y is the concentration of substance in storage, and V is the volume of water in storage, then equations (1) and (2) become

$$\frac{d(Vy)}{dt} = q_I C - q_R y - \text{decay rate} \tag{3}$$

$$\frac{dV}{dt} = q_I - q_R \tag{4}$$

Furthermore, if releases of water from storage are made proportional to streamflow, S, and inversely proportioned to y, if possible, without exceeding the storage capacity, V_{max}, or without reducing storage below some minimum level, V_{min}, and if releases are set equal to q_Z at those limiting conditions, then outflow is given by the following relationships

$$q_R = AS/y \text{ for } V_{min} < V < V_{max}$$

$$q_R = q_I \text{ for (a) } V = V_{min} \text{ and } q_I \leq as/y \qquad (5)$$

$$\text{and (b) } V = V_{max} \text{ and } q_I \geq \frac{as}{y}$$

where a is the constant of proportionality.

Relationships for decay rates and time dependence of the environmental variables, S, q_I, C and temperature, T, must be specified to complete the mathematical model. Such relationships must be determined from observations on the real world by a process described as follows in a more general context.

Let the true real world process that is to be modeled be represented by the expression

$$\zeta = \Gamma (N_1, N_2, \ldots , N_m)$$

in which ζ is the variable or response that is to be predicted, $(N_1, N_2, \ldots, N_m)$ is the set of variables upon which ζ is dependent, and Γ is the function representing the true causal relationship between ζ and the N's. Scientific research may be viewed as a search for that set of N's on which ζ is dependent and for the function Γ that specifies how those variables are related. The search proceeds by observing a tendency for certain variables to vary in a more or less consistent manner. Let Y and $(X_1, X_2, \ldots, X_R)$ be a set of variables that may be observed and are taken as representative of ζ and $(N_1, N_2, \ldots, N_R)$. Let y_i and $(X_{1i}, X_{2i}, \ldots, X_{ki})$, $i = 1,2, \ldots, n$

be a set of n observations on Y and $(X_1, X_2 \ldots X_R)$ respectively. Variables Y and X may be either the ζ and N's respectively, operation definitions of the latter variables, or even surrogates for those variables. In any case the observations y_i and X_{ij} may contain errors assignable to limitations, either in definition or measurement. Nevertheless, from this set of data and other knowledge, a relationship of the form

$$y_i = g(X_{12}, X_{21}, \ldots, X_{ki}) + C_i \text{ may be suggested}$$

in which g is taken as an approximation to Γ and C_i represents the discrepancy between y_i and that value that is predicted by the function g and its arguments. Although even the "best" data is subject to error, it constitutes the extent of available information on how the real world behaves and the search continues for the "best" model or functional relationship that in some sense minimizes the discrepancies C_i. The process, according to Bross (1953), proceeds as shown in Figures 1 and 2. Beginning with observations on the real world, a symbolic (mathematical) model containing unspecified parameters, is constructed, parameters are estimated, and the symbolic model is manipulated to yield predictions about the real world. These predictions must be compared with new field data in order to evaluate the quality of the model. As in Figure 2 the process is depicted as being iterative in character. Unless one is dealing with reasonably well understood phenomena, there is little reason to believe that the first symbolic model to be suggested will be the best from among several alternatives.

What bases exist, then for suggesting a particular functional form that specifies a relationship among variables? There would appear to be numerous approaches to solving this problem, including (1) borrowing upon the recorded experience of others, (2) graphical representation, (3) intuition, and (4) guesswork. In the absence of other knowledge, a fruitful criterion has often been that of choosing the most analytically tractable model that yields a satisfactory approximation. This criterion has often lead researchers to the adoption of linear and polynomial approximations, which are among the most tractable of all mathematical relationships. Extraction of those approximations from data sets has been considerably aided by formal methods of statistics and numerical analysis.

The above argument can be applied to obtain a model of decay rates for the substance in storage. Although such models are strictly descriptive in the same sense that the model reveals little about causal mechanisms, decay rates that are sensitive to temperatures have often been given by the relationship

$$\frac{dy}{dt} = rb^{(T-2C)}\,y^{j} \qquad (6)$$

where the integer j is the so-called order of the process, V is the constant of proportionality, b is some base, generally slightly greater than unity, and the temperature T is given in degrees centigrade (temperatures of water in storage is assumed to be approximately the same as ambient temperatures). If on the basis of experimental data, we determine j to be 2 (a second-order process), then equations (3), (4), (5) and (6) can be

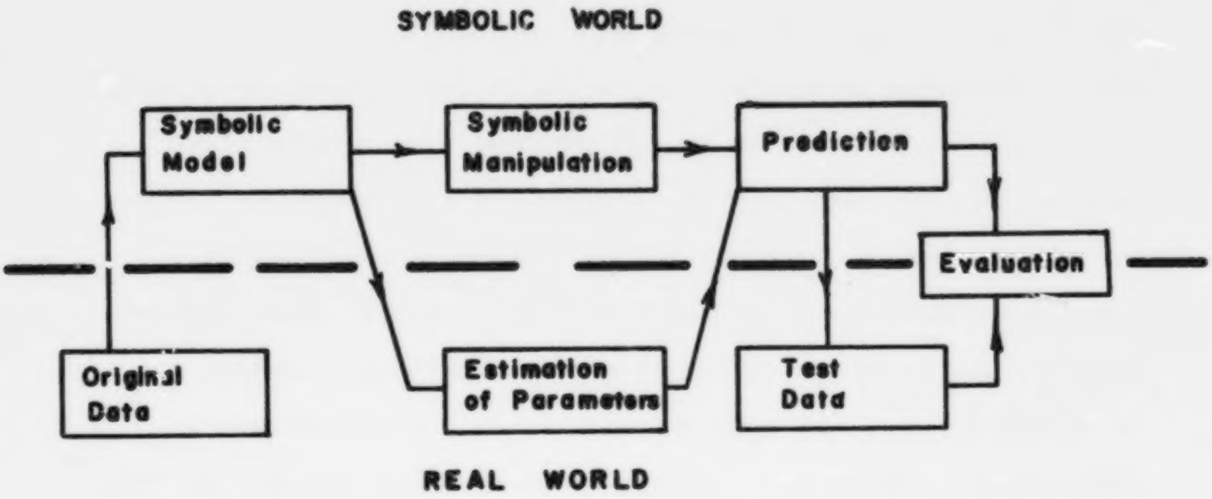

SYMBOLIC WORLD
Symbolic Model
Symbolic Manipulation
Prediction
Evaluation
Original Data
Estimation of Parameters
Test Data
REAL WORLD

Fig. 1. PROCESS OF MATHEMATICAL MODEL CONSTRUCTION
(After Bross)

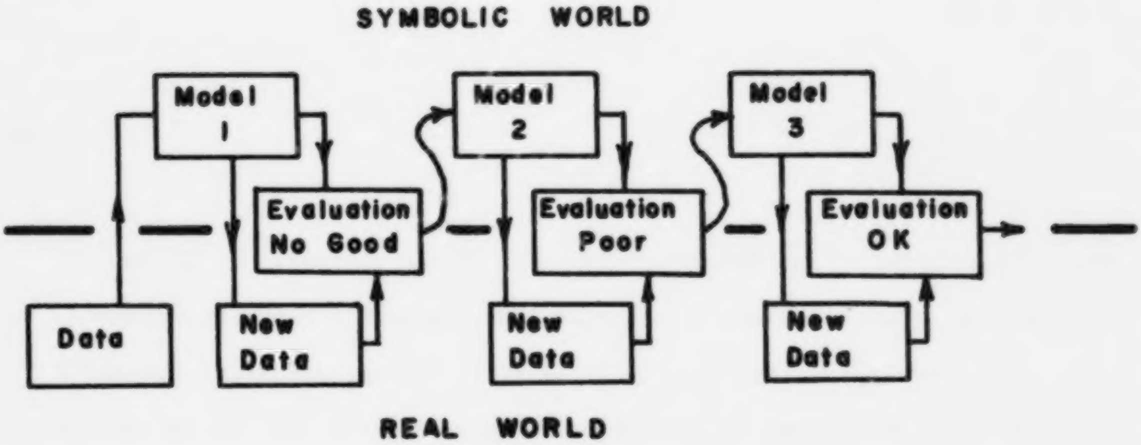

SYMBOLIC WORLD
Model 1
Model 2
Model 3
Evaluation No Good
Evaluation Poor
Evaluation OK
Data
New Data
New Data
New Data
REAL WORLD

Fig. 2. ITERATIVE PROCESS OF MODEL CONSTRUCTION
(After Bross)

combined to get

$$\left.\begin{aligned} \frac{dy}{dt} &= \frac{1}{V}\left[(C - y)q_I - kb^{(T-20)}\, y^2\right] \\ \frac{dv}{dt} &= q_I - as/y \end{aligned}\right\} \text{for } V_{min} < V < V_{max} \qquad (7)$$

and

$$\frac{dy}{dt} = \frac{1}{V}\left[(C - y)q_I - kb^{(T-20)}\, y^2\right]$$

$$\begin{aligned} &\text{for either} \quad V = V_{min} \text{ and } q_I \leq \frac{as}{y} \\ &\qquad\quad \text{or} \quad V = V_{max} \text{ and } q_I \geq \frac{as}{y} \end{aligned} \qquad (8)$$

The mathematical model now consists of two simultaneous differential equations that relate the two endogenous variables y and V when V is not at a limiting value, and a single equation for the case where V is fixed at one of the limiting values. Also expressed in the model are the relationships among the endogenous variables and the exogenous variables q_I, C, S and T. Those expressions are solvable for the time dependence of y and V using methods of mathematical analysis but before doing so, he must make certain decisions about uncertainties in the description of the system and about environmental variables.

UNCERTAINTY

In the context of mathematical descriptions of systems as outlined above, five sources of uncertainty may be identified. First, the set of variables (N_1, N_2, ..., N_m) upon which the response ζ is dependent may be incorrectly or incompletely identified by the choice of variables (X_1, X_2, ..., X_e) that are included in the model. Second, the functional form of g which relates Y and X in the model is an approximation to the true relationship Γ. Third, Y and X may be only surrogates on operationally defined variables that are assumed to be representative of conceptually defined variables in the real world process, and observations on Y and X contain measurement errors. Fourth, parameters, such as rate constants in equations (6), must be estimated from error-contaminated data, and finally, the fifth source of uncertainty lies in the random components of time series models that describe the time dependence of environmental variables. Except in the latter instance, uncertainty may be reduced through investments in further research, improvements of instruments and techniques of measurement, and additional observations on real world processes. Indeed, the process of reducing uncertainty in the description of systems is continual and evolutionary, but at any point in time, if models are to be constructed, they must be constructed with uncertainty about the system and its environment. Additional efforts can only reduce uncertainty, they cannot eliminate it.

An analyst may choose either to ignore uncertainty and treat the resulting "deterministic" problem where no chance variability is considered or he may choose to explicitly model uncertainty giving rise to so-called "stochastic" models. Analyses of deterministic problems are generally less demanding of mathematical competence as developed in traditional educational processes and are quite useful when uncertainties are believed to have negligible influence upon the solution. In those cases, errors may be neglected on the basis of accumulated experience with similar systems or upon formal sensitivity analyses when effects of uncertainty on the solution are quantitatively evaluated. Where uncertainty cannot be ignored, random error terms may be added to mathematical models of either the system, or of its environment, or to both. Resulting solutions to the system of equations that define the system and its environment must then consist of probability functions that define responses.

Variations over time of environmental variables are themselves determined by other natural or man-made systems, but, for the purpose of analysis of a single system as illustrated here, it is assumed that their variability can be summarized by exogenously determined functions of time. Such functions may be determined by methods of time series analysis in which a variable ζ is generally assumed to be a linear combination of some long term trend, $\tau(t)$, a seasonal component, $\sigma(t)$, and a random process $\rho(t)$, so that

$$\zeta(t) = \tau(t) + \sigma(t) + \rho(t)$$

Statistical analyses of time-series may be employed to estimate parameters and test the "goodness of fit" of alternative functional forms for τ, σ, and ρ to a time stream of observations on the variable ζ (1966).

If time series models for the environmental variables q_I, C, S, and T are found, then the pollution control system is described by a set of mathematical expressions that may be subjected to analysis to examine the response of concentration, g, volume, V, and more importantly, the releases of pollutant to the stream as those variables respond to controllable and uncontrollable factors in the environment.

SOLUTIONS OF EQUATIONS

At least three distinctly different approaches to the solution of equations or systems of equations have been developed for determining the time dependence of endogenous variables. These include methods of real analysis, numerical analysis, and simulation. While distinctions between real and numerical analysis may be drawn quite easily, simulation (1966) is a loosely defined area of analysis that draws heavily upon results of both real and numerical analysis and upon methods of statistical analysis to experimentally determine the response over time of endogenous variables that are related by some mathematical models that may either: (1) defy assumptions upon which other solution methods are predicated, (2) be too complex for analysis otherwise, or (3) be a less expensive method than its alternatives.

In lieu of defining and classifying the three areas of analysis, examples can be cited to illustrate contributions of the alternative approaches. In real analysis, solutions to general linear, quadratic, and cubic algebraic equations are well known. Formulas have been found for the zeros of these polynomials; formulas in which the zeros are specified simply by substitution of appropriate constants. Similarly, methods are well established for the solution of many linear differential equations. For example, if the decay process was assumed to be first order and the volume was held constant, equations (3) would become a linear differential equation with the following solution:

$$y(t) = \frac{1}{p(t)} \int_0^t \frac{1}{V} p(x)C(x)q_I(x)dx + \frac{K}{p(t)}$$

where K is some constant to be evaluated when t is zero and the function p(t) is given by

$$p(t) = \exp \{ \frac{1}{V} \int [kb^{(T(t)-20)} + q_I(t)]dt \}$$

In this example, q_I, C, and T are assumed to be known functions of time with negligible uncertainty. Indeed, if the problem was of sufficient importance and generality to merit considerable attention to the nonlinear mathematics, exact solutions to equations (7) and (8) might be found. For this problem, the effort seems hardly justified, especially since alternative methods of solution are readily available. The second alternative is numerical analysis. Here it may be noted that if q_I, C, S, and T are known, continuous differentiable functions of time, then nonlinear equations (7) and (8) may be solved to yield numerical values of y and V, to an arbitrary precision (except for round-off errors), at any value of t. Runge-Kutta methods for two simultaneous equations may be applied to equations (7) until the volume reaches either of its limiting conditions, at which time, the solution may be advanced by similar methods for a single equation (8). This solution will be valid until outflow can once again be set equal to as/y.

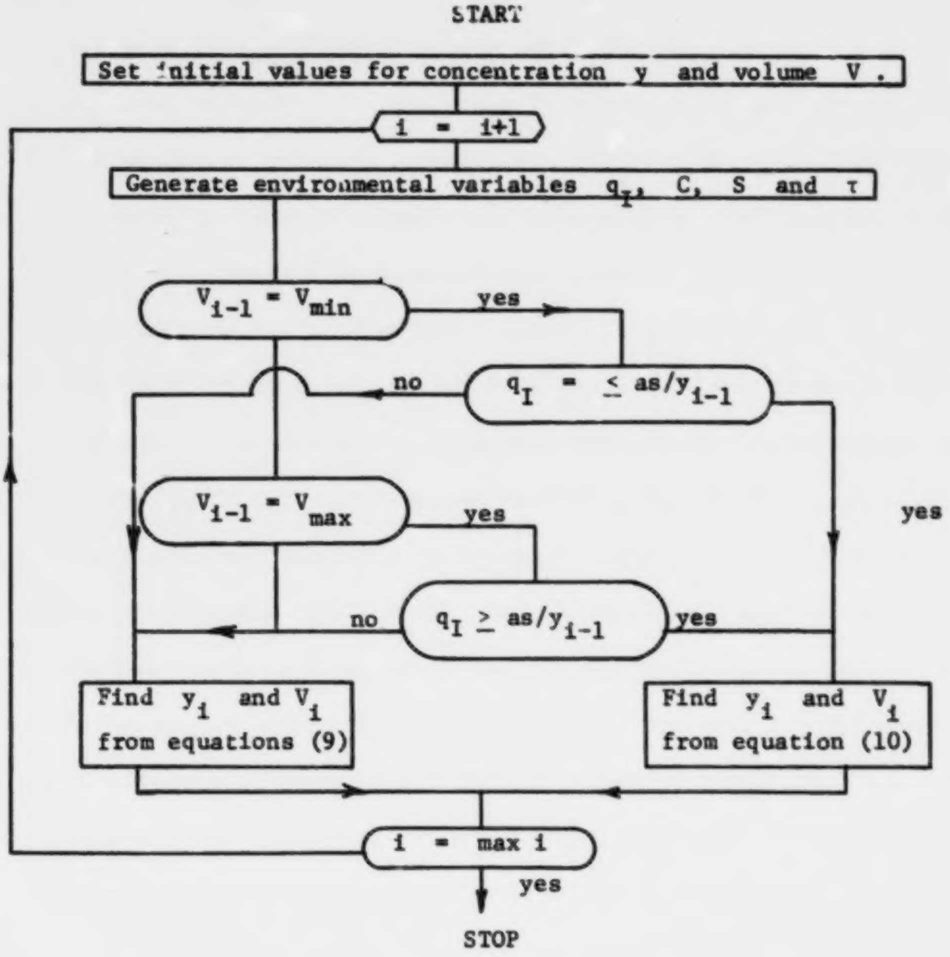
Finally, if q_I, C, S, and T are assumed to be arbitrary functions of time containing stochastic elements, the most readily available method for treating the stochastic-nonlinear system defined by equations (7) and (8) is through Monte Carlo simulation. Briefly, the model can be constructed as follows. Divide real time into discrete intervals of width Δt and write the finite-difference approximations to equations (7) and (8):

$$y_i = y_{i-1} + \frac{\Delta t}{V_{i-1}} \left[(C(t^*) - y_{i-1})q_I(t^*) - kb^{(T(t^*)-20)} y_{i-1}^2\right]$$
$$V_i = V_{i-1} - as(t^*)\Delta t / y_{i-1} \quad \text{for} \quad V_{min} < V_{i-1} < V_{max} \tag{9}$$

and

$$y_i = y_{i-1} + \frac{\Delta t}{V} \left[(C(t^*) - y_{i-1})q_I(t^*) - kb^{(T(t^*)-20)} y_{i-1}^2\right]$$
$$\text{for either } V_{i-1} = V_{min} \text{ and } q_I(t^*) \leq as(t^*)/y_{i-1} \tag{10}$$
$$\text{or} \quad V_{i-1} = V_{max} \text{ and } q_I(t^*) \geq as(t^*)/y_{i-1}$$

where subscripts i and $i-1$ indicate values of the variables evaluated at the end of time periods $(i-1)\Delta t$ to $i\Delta t$ and $(i-2)\Delta t$ to $(i-1)\Delta t$, respectively, and t^* is taken as the mid-point of the interval from $(i-1)\Delta t$ to $i\Delta t$. Each of the values for q_I, C, S, and T are assumed to be given as the sum of some deterministic function of time and a randomly generated value from some stochastic process. A flow chart for the solution process is given in Figure 3. Application of that computational scheme will yield one of many possible time streams of values for y, V, and releases.


START
Set initial values for concentration y and volume V .
i = i+1
Generate environmental variables q_I, C, S and τ
V_{i-1} = V_min
yes
no
q_I = ≤ as/y_{i-1}
V_{i-1} = V_max
yes
yes
no
q_I ≥ as/y_{i-1}
yes
Find y_i and V_i from equations (9)
Find y_i and V_i from equation (10)
i = max i
yes
STOP

(FIGURE 3.)

Repetition of the scheme will produce several possible series of responses for those endogenous variables, the properties of which may be summarized by methods of statistical analysis.

SUMMARY

Thus, in summary, mathematical models can be employed advantageously to deduce the behavior of certain variables that are endogenous to a system as that system is subjected to environmental stimuli. The process consists essentially of two parts, describing the system in mathematical language and analysis of the resulting system of equations. Description involves the isolation of the system from its environment, applying equations of continuity to account for the flow of energy, mass or other objects of interest through the system, finding relationships to indicate the rates at which those flows occur, and establishing relationships between endogenous variables and those in the environment, including the rates at which the system exchanges energy, matter, and information with its environment. Several alternative methods are available for analysis including real and numerical analysis and simulation. Advances in the field of computation have freed analysts from the demanding tasks of solving equations by advanced methods of analysis and from some of the limitations of those methods. This freedom permits a more realistic formulation of problems, including the use of nonlinear-stochastic systems. Having this freedom of analysis to deduce consequences of making decisions about the levels at which certain parameters in the environment of the system are set, the pressing tasks before the scientific community are to improve descriptions of real world processes and examine the complex questions of evaluating those consequences that may be deduced from mathematical models of specific systems.

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A FURTHER NOTE ON VERIFICATION*

The following note concerns model "verification" and the relationship of mathematical models to the real world. It is intended that this note will complement the discussions of this topic that occurred during the first sessions of November 19-20.

The role of mathematical modeling in the understanding of the real world is schematically shown in Figure 1.

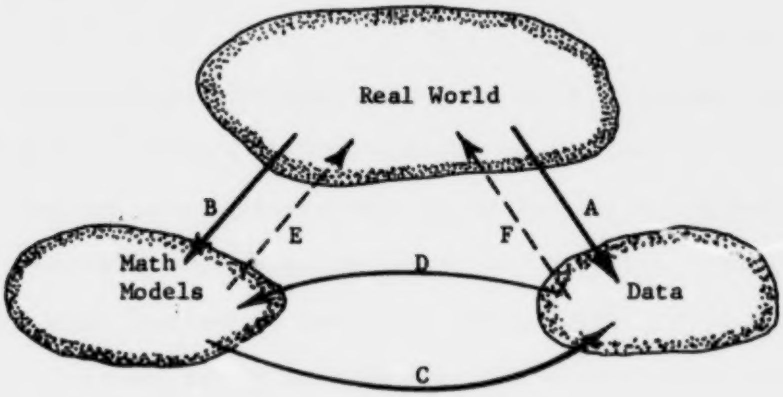

Figure 1 - Conceptual Diagram

In this simplified diagram, three conceptual realms are shown: the real world, mathematical models, and measurements or data. The investigator is interested in solving problems within the real world and uses the mathematical models and data to gain the understanding necessary for the solution of

*by David A. Bella

real world problems. An investigator may collect data from the real world (line A) and/or he may construct mathematical models of the real world (line B). The experimental procedures used to collect the data may be simple or complex, as may be the mathematical model. One cannot deny that data can be taken (line A) and that models can be developed (line B), although both the data and the models may or may not be worthless.

One can also use the mathematical models to generate data (line C) or one can fit the data into the mathematical model (line D). All steps shown by lines A, B, C and D can be performed yet none of these steps lead back to the real world.

Mathematical models and data are greatly simplified abstractions of ideas of the real world taken and developed by limited and biased investigators. To gain insight of the real world, the investigator seeks some order from these abstractions and from this order makes some judgements concerning the real world. That is, the steps back to the real world involve a judgement process (thus lines E and F are shown as dashed lines).

In the field of water quality models, one often hears of models being "verified". Model verification by definition means the proof of the truth of the model. Such a definition, however, in no way describes the process of relating the mathematical model results to the real world. The words "proof" and "truth" imply far more rigid requirements than is possible for relating the model to the real world. Rather, an investigator must justify the conclusions reached from the use of the model for each particular set of conditions in which the model is used.

Too often a model is "verified" by attempting to demonstrate that the data and model results agree (often nothing more than a closed loop shown by

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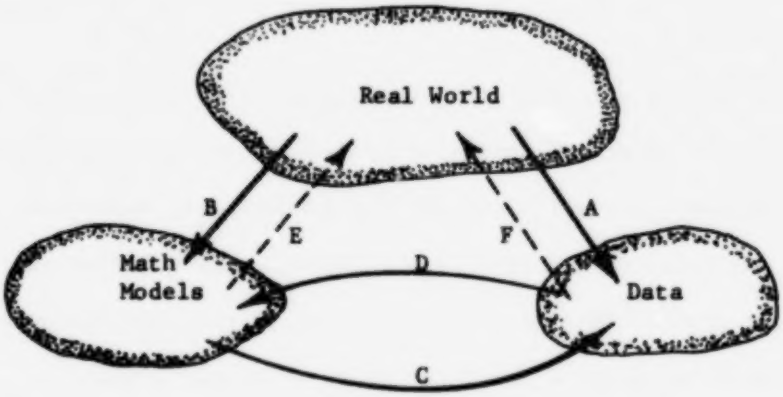

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Too often a model is "verified" by attempting to demonstrate that the data and model results agree (often nothing more than a closed loop shown by lines C and D). Such attempts are typified by a plot of "theoretical" predictions closely winding its way through a maze of "measured" results. Such a plot can demonstrate that the investigators have:

1. committed the same errors in collecting the data as were committed in constructing the model,
2. successfully fitted the model to the data, and/or
3. successfully fitted the data to the model.

An example involving all three of the above list (the list is not limited to three) might result from a completely mixed DO model of a stratified lake. The model, of course, can generate many types of data, depending on the parameters chosen. During the summer months, the DO in a stratified lake often varies from supersaturation in the surface regions to zero (or near zero) at greater depths. One can thus collect many types of data trends by "properly" selecting the depth. Variable depth sampling would be particularly useful for such model "verification" as it allows the investigator to better fit the data to a menagerie of "theoretical" results.

Agreement of measured and theoretical results can, however, serve as evidence that the use of a model in a particular set of conditions leading to a particular set of conclusions was justified. To support this evidence, the investigator must define the limitations of and dependency (or lack of dependency) between the model and the data. Often, other evidence might be even more effective. As an example, data which shows that the principal assumptions within the model were reasonable for the particular case in which the model was used might be effective evidence. An effective discussion of model results in which the causal relationships expressed within the model are logically presented will also support conclusions reached through the model use.

In short, a model cannot be verified but rather the use of the model is justified for each conclusion reached through the model use for each situation in which the model is used.

ACKNOWLEDGMENT

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DISCUSSION

CONCEPTS OF MATHEMATICAL MODELS

PATTEN: What do you mean by the central structure of a system, in general words, a vertical structure, a vertical order, or just what does the term mean?

MOREAU: The various processes tied together by a central natural process.

BURNS: Talking about a model for a specific purpose, if you are going to model something, take the water resources of the cities. I think we are exposing ourselves to a complexity we can't cope with at that level. Any model we talk about must aggregate in some sense many of the variables in nature, or our own social systems into classes, that we can pragmatically cope with.

PATTEN: So the central structure is that, and not the structures of the real world.

MOREAU: Two different people may devise different structures, and not model the same way.

PATTEN: What you are saying is, the system has a structure and you are pointing out certain features to find out the answer to your problem.

MOREAU: We are trying to abstract that problem from all we have available from the real world.

DUTTWEILER: Our understanding of the real world is based on data gathered for a specific purpose, so you are really not modeling the real world, but your representation, your objective understanding, perhaps, of the real world, so there are really not two, but three, perhaps, criteria in the modeling phases. So there is a real world, and then a quasi-real world, and a symbolic world.

SCHAAKE: You used the notion of the symbolic world and the real world. In order for symbols to have meaning, they must be interpreted in a particular context. This is true in the output of your computer program. There are lots of symbols there, but they only mean something in terms of the context in which they are done, and this relates essentially to the same point. The context in which something is accomplished gives it the meaning that it has. Whatever meaning attaches to the symbolic world, it attaches to it because of the context.

PATTEN: You haven't justified the mathematical models. You haven't spoken about other kinds of models, and I am wondering why the emphasis on the mathematical models.

CHEN: We cannot purposely eutrophy a lake in the prototype. We have some experience in working with mathematical models against physical models, and we found out that the mathematical modeling is much more efficient than the physical model and much easier to develop.

MOREAU: There are two kinds of reasoning processes: There is what I would refer to as an inductive process, which draws from our observations on nature. We abstract from these observations a description of how nature performs and how it behaves. We also have the deductive process, in which we go through a logical sequence of steps in the abstract leading to results that we cannot create solely from observations.

PATTEN: Doesn't this stem from your mathematics, and from the fact that once you get your real system represented mathematically you can sort of turn the crank. I mean, there is a well-developed crank determined and you can generate output automatically.

MOREAU: I think we tend to think of that as a well-developed crank. People have developed new forms of turning the crank, and in fact developed new kinds of mathematics. I think the development of calculus is a very good example here, as far as geometry. . . the statements that were available to Newton in expressing this change, the rates of change, simply were not adequate to the task, so he developed a new way of turning the crank. There have been some rather fundamental developments in mathematics in recent years. I think we have to think of these two kinds of processes not only as some kind of dynamic process in which the mathematics may be out of equilibrium with the physical sciences at times. It may be that mathematics at times may lead the physical sciences. We are in a sort of dynamic process in which we are searching for new ways of questioning things.

PATTEN: So what have we got in modeling? We have types of things - mathematical and physical.

MOREAU: We have the kinds of models in which models look like a prototype, scale replicas. Then we've got the allied models in which you use properties of models that represent different kinds of properties of the prototype. Some models, of course, are a combination of these things. The properties of hydraulic models have principles of similitude. Again, we know that there are several different kinds of forces operating in nature. You can build a hydraulic model, for example, that represents what you have in scale replica. You can also make it reproduce the ratios of two different forces operating in nature. You can't make your replica maintain all of the similitudes. You can't make it preserve all of the ratios of forces. So you still, even with those kinds of modeling, you've got to find out what the dominant forces are.

VOLLENWEIDER: I feel that in this kind of discussion we accept two things: we have on one side scientific research accumulated now from many areas. We know exactly that nitrogen, phosphorus and reformulated protein in the water, recycling, and so on, is important. This research has to describe quite clearly a use for this research in the kind of language which we use. There is a tremendous amount of repetition of what we already know. We make qualitative statements, and put this into a computer. What we have to know is fundamentals if the model building has any value. We have to ask ourselves if we increased recycling what is the real quantitative reaction of the system.

CHEN: You don't describe what you want, either. This is the problem. I think our model has to lend itself to whatever kind of problem you are trying to solve. The function of a model is not to discredit whatever we learn from fundamental understanding. The purpose of the model is to try to bridge the gap, to bring the fundamental understanding into a possible prototype. There are so many ways to do this. This type of question cannot be answered by purely fundamental research.

FRUH: Going back to one of your model concepts, suppose you were wrong in one of your equations.

CHEN: It is not me. If I am wrong, then the literature is wrong.

FRUH: In your concept of what the literature is, there are some equations up there that many of us would have doubts about. If your sensitivity analysis changes your case, you can check out which is important to your sensitive case, within your equation, but suppose your equation - we are back to concept again - but what you think your computer models of the real world, is wrong, the thing we have to go back to now is trying different concepts, and not just the one equation, and then go on to the sensitivity analysis.

CHEN: You have to understand that through this sensitivity analysis we try to discern why this is so, then eventually the model has been closely reworked and so on.

MOREAU: You could think of this in the sense of Newton's laws of motion. His model of the universal laws of gravitation was the subject of his empirical statement of how he inferred the law would behave, and he wrote this down in mathematical terms. It represented his best thinking at that time. Obviously it explains a lot about what goes on, and it took a couple of hundred years for Einstein to come along and find the error in his logic. Obviously nobody has a corner on truth. These are simply symbolic representations of what we understand about the real world. To make the point, there are the two processes. First, the inductive process in which you try to describe what's going on, and if your description is no good, if your basic postulates are no good, obviously you are playing a nonsense game. But the question is also, "How good?" We don't reject Newton's laws of motion today certainly because they don't account for the most minute kinds of variations, because most of the problems originating in the empirical world are very well explained by Newton's laws of motion. They are still very valid explanatory models of what's going on.

BURNS: In talking about the objectives of eutrophication studies, it seems that one thing we are not really stressing must be looked at very intensely. It is the interface of man's society with the eutrophication process. Certainly man has inputs into the system in terms of the nutrient additions to the volume of water in terms of recreational and commercial uses of that water, and has outputs in terms of the effect upon that body of water. The point is, day to day specifically, what is the danger of eutrophication? As a member of society the water looks fine to me. What are the warning implications of eutrophication? Is this something that we really can be concerned about? If you can convince me of that, then maybe we can work backwards to see what aspects of eutrophication, of the biological-chemical pollution properties must be operated on.

BEETON: I want to make a comment relative to what Burns brought up about how you define bacteria, and so on. I think that some of us are expecting too much of the modeling program. I think of modeling as defining the kinds of questions I should be asking. I think it is almost implied that we will come up with some answers. I think the real danger of it is that we are expecting too much of it. I prefer to use it as a tool in my research to find the kinds of things I must pursue.

BELLA: We have to make decisions now, and we have to use the tools that are available now even though they are developing. We will be using models to make decisions, even though we are very well aware of the limitations.

MOREAU: There are indeed several different kinds of objectives to be served in the model building process, some relating to decisions, some relating simply to descriptive kind of modeling to assess the state of the system at any given time. We will talk about these a little later in the program. We also have the models that are essentially related to the understanding of the essential structure. I don't think we can cast all of the different types of models in one or another mold. I have suggested three. You may wish to add others to the categories later.

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MODELS OF AQUATIC ECOSYSTEMS

A Panel Paper

by

B. C. Patten

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INTRODUCTION

Computer models, or more specifically computer implemented mathematical models in planning, science, and engineering, are rapidly finding many practical applications. For many years there has existed in most professions a hard core of people who have used these models and who have advocated their usefulness to other. Growth of their use has probably been exponential, and it now seems that we are entering a new phase where legitimate uses of mathematical models will more easily be accepted. Early prejudices are to be replaced with a kind of restrained objectivity.

One reason for optimism about the future of mathematical modeling is that both the potential and the limitations of these models are becoming better understood with increasing experience over a wide range of disciplines. Major new developments require time and experience before many people can appreciate their ultimate usefulness. Also, promising new ideas for solving important problems usually can find modest financial support. This attracts people who may or may not be competent to develop the ideas, so there are always many bad examples to cite in rebuttal to proposals that the new ideas have merit.

The underlying purpose of many mathematical models is to give insight, rather than to give exact numerical values that predict some future state of the system. This is a key point. While there are some systems that can be modeled with great accuracy, such as the solar system, and the objectives of these models may be to obtain very accurate information about these systems, there also are systems such as aquatic ecosystems where the apparent "noise component" is very large. In these systems, exact solutions are not possible, so they should not be sought as the purpose of the model.

GENERAL RATIONALE FOR MODELING AQUATIC SYSTEMS

A model of an aquatic system--or any system--is an abstraction, a simplified representation of the system, and not a complete or exact duplication of it. This simplification of nature, as we shall point out, is the source of both the value and the weakness of the modeling approach to ecological problems. The model presents a complex system in a sufficiently simplified form that it can be grasped by the mind and the operation of the system simulated on a computer. Simulation provides a means for reconstructing past states and predicting future states of natural systems. The economic importance of modeling is derived from this latter proposition. A suitable model should be able to answer the questions, "If man does x, what will nature do?". The complexity of nature is such that in the current absence of suitable models these questions are either ignored, or approached on a costly trial-and-error basis.

The weakness of the modeling approach is that a model, like a map, is an incomplete representation of nature. Just as no one map can reveal every detail of geography, no one model can simulate every aspect of a natural system. No one model can answer every question which might be asked concerning a natural system. It is thus unlikely that an arbitrarily selected model, even though a marvelously precise homolog of the natural system, will prove sufficient for the solution of a given practical problem. The prime consideration in constructing a model is therefore the purpose to which the model is to be put!

Aquatic ecosystems are usually well defined and the number of interacting processes that regulate their activity is very large. In the past, coordinated field and laboratory investigations have served as the primary basis for understanding important relationships. In modeling, formulation depends in large part on the assessment by a given investigator and his co-workers of what constitutes the significant elements of the system. This is subject to an additional constraint dictated by the present state of our knowledge which requires that we solve a modeling problem piecemeal, that is, by building and testing submodels with the hope that they will be compatible and together will generate "good" simulation results.

The strategy for planning a mathematical model is not much different from the strategy for planning, in general. Fundamentally, there are four basic steps in the process. These are:

1. Carefully identify and evaluate the objectives
2. Select criteria for meeting these objectives
3. Seek the best of all possible plans that satisfy the criteria
4. Evaluate consequences of the plan

These steps are not always followed, but there probably is a high correlation between successful plans and those which followed these four steps.

The distinction between objectives and criteria (i.e., between steps one and two) is very subtle, and subject to different interpretations. The first step is the most important one and also the one most often overlooked. In planning a mathematical model, the person or group that is to use the model is free arbitrarily to state objectives. The results of subsequent work should be judged solely in terms of how well the objectives are satisfied. Objectives should often be broadly stated because there frequently is more than one way to satisfy most human objectives.

After the objectives have been stated, criteria for meeting these objectives must be selected. This is a difficult step. In group situations, agreement or consensus on criteria may be difficult to achieve because more than one set of criteria may relate to the same objective. Finally, a given set of criteria may be difficult to satisfy, or else the best possible plan may be impossible to identify.

Another reason why objectives and criteria are often confused is that objectives may seem so obvious that they are taken for granted and not clearly stated. For example, most public water resources projects should be planned to achieve the best possible social welfare benefits. This obviously must account for economic costs and benefits which must be included in the criteria. However, it is often stated that economic efficiency is the objective. This is not true!

A final reason for confusion is that the ways people do things become established over time so that objectives may get confused with the way things have always been done. There is nothing wrong, necessarily, with doing things the same way, but this should not be confounded with the objectives that led to doing them that way. Confusion between objectives and the way things are done may have very subtle consequences. For example, people intimately involved in their work may have difficulty explaining to someone else what they do. Some of us finally seem to lose touch with the original objectives that led us to do our particular thing. When that happens, it is almost impossible to communicate about our work.

The third step of the planning procedure is to choose the best possible model to meet the criteria. This involves choosing from among the possible kinds of models that might be appropriate. In general, the simplest and easiest function model that satisfies the criteria should be used.

The final step is to evaluate the consequences of the model. This includes verifying the model and showing that the objectives have, indeed, been met. In the case of management models, which are used for decision-making purposes, this step should also include evaluating any external effects that may result from actions indicated by a particular set of decisions.

PURPOSES AND OBJECTIVES OF WATER QUALITY MODELS

The purpose in developing water quality models is the solution of practical problems relating to utilization of water supplies and the handling and treatment and sewage and other waste, for example, setting up criteria for acceptable levels of phytoplankton nutrients in sewage effluents. The objective in the construction of water quality models is thus the development of efficient tools capable of producing the needed information with suitable precision and accuracy on the basis of a minimum input of data. Just as a machine should have no unnecessary parts, these models should contain no flows, compartments, processes or relationships unnecessary to their purposes. It is likely that useful and functional water quality models can be developed without knowing everything about the ecology, chemistry, and physics of the natural systems!

This approach does not preclude the development of models which extend our theoretical knowledge of natural systems. Detailed and extensive models of this type might prove to be the simplest models which can provide needed information. Comprehensive models may also be needed to determine which flows or compartments can be deleted or combined in a final water quality model. The development of such models should, however, not be considered equivalent to the construction of water quality models.

A water quality model dealing with eutrophication should be thought of as an experimental tool designed to shed light on the nature of complex input-output relationships in a particular aquatic ecosystem. This implies incorporation of productivity functions that have their origin in the field and laboratory. This model will then become an interface for ideas, yielding results that suggest areas for further investigation. Subsequent evolution of the model utilizing information obtained from these new studies may gradually provide the predictive capability needed to compare management strategies. The ultimate goal should be environmental control designed to optimize system output in the sense of satisfying as many segments of society as possible.

A common goal for water quality models is to explain or to reproduce water quality response. This is not a valid goal because it is impossible to set adequate criteria. Unless the model becomes the actual aquatic system, there always will remain some uncertainty in our ability to account for system response. To make this goal valid, the purpose for which the model is to be used must also be stated.

For example, some water quality models are used in a management context where investment decisions are to be made in water quality control facilities. In this case, the purpose of the model should be to assist in this decision-making process. Such models should not be judged in terms of their ability to represent water quality response. Rather, they should be judged on the basis of whether the marginal value of additional information from an improved model is in balance with the marginal cost of improving the model. In this context it is possible to develop too good a model because too much money was invested in the model.

Another purpose for water quality models is to identify the forces or phenomena that cause the system to respond in some particular way. Development of these kinds of models is an important part of scientific research. These models are important because they allow us to classify and store our knowledge. This makes it possible to say without further experiment that certain phenomena generally behave in certain ways. These models also make it possible to synthesize knowledge and to anticipate the behavior of systems before it is possible to observe them. Such models should be designed so there is a very high degree of confidence that the "correct" choices will be made as to the forces or phenomena that underlie the system. These models are used as part of the scientific method to test hypotheses that certain statements or theories about the system are true.

KINDS OF MODELS APPROPRIATE

In the preceding sections, two general modeling categories were identified: (i) management models and (ii) scientific research models.

The role of water quality modeling in management models is to show the relationship between decision variables and the consequences of those decisions. For example, in managing the water quality of, say, Boston Harbor, one of the water quality targets could be to prevent the occurrence of sea lettuce along mud flats. A model is needed that shows how sea lettuce responds to these decision variables. There must be more than one way to model the response of sea lettuce growth to decisions either to treat municipal wastes or to reduce combined sewer overflows. Where enough data exists, statistical models could be built. However, the validity of these may be suspect unless it can be argued that the relationship accounts for the phenomena which cause the system to respond.

Models may be regarded as deterministic or stochastic, continuous or discrete, or (more typically) several types in combination. In general, a model would normally originate as a deterministic one and stochastic features should not be introduced until after its behavior is understood over a reasonable range of parameter values. It is likely, however, that both will be necessary if the final model is to approach reality. It also seems inevitable that both continuous and discrete variables must be employed to adequately describe most real systems since virtually all realistic models are intractable mathematically when one is working at the ecosystem level. This usually means that Monte Carlo techniques and sophisticated numerical methods are essential for useful results to be obtained.

Dynamic ecosystem models are most often constructed out of systems of simultaneous differential equations. These equations may account for the transfer of nutrients from one trophic level to another, or they may account for the physical movement of nutrients in the same trophic state from one location to another. The equations may involve ordinary and partial differential equations. They are likely to be nonlinear, but the value of linear approximations should not be overlooked. Difference equations and differential-difference equations may also be useful.

Ecosystem models should be studied both from physical and mathematical viewpoints. From the physical side, the meaning of various terms in the models may be argued. Any ecosystem model is a conceptualization of the real world, so the terms in the model can only represent an imperfect concept of the real world. As a result, these terms may give rise to parameters which are not easily measured by any single chemical or biological test. Some of these parameters may be measurable only by inference from the total system response.

Mathematical studies of ecosystem models are required for several reasons. First, there exists now a large body of knowledge about the general behavior of linear models. This existing body of knowledge, when applied in studies of linear ecosystem models, can be used to demonstrate that these models have certain response characteristics. If the linear model is valid, then the ecosystem may be anticipated to have these characteristics as well. Nonlinear models have not been well studied in general although there are some very well documented ones. As different groups develop nonlinear ecosystem models it will be useful to study the mathematical response properties of these models.

Another need for mathematical studies of ecosystem models is to investigate the stability of parameter estimating techniques that rely on total system response to estimate parameter values. The simpliest form of model in which parameters are estimated on the basis of the total system response are regression models and other multivariate statistical models. More involved models involve time series techniques such as those used extensively by economists for analyzing economic time series. These kinds of models have special mathematical properties, and these properties may be inferred to exist or not exist by the modeling technique used to model aquatic ecosystems. The most complex form of model are dynamic models based on simultaneous differential equations. Parameters in these models may be estimated by response surface optimization techniques or by inference from special characteristics of system response.

Two of the three general model types identified by Levins (1966, Am. Sci. 54: 421) appear most appropriate for water quality models. Models which "sacrifice precision to (gain) realism and generality" may be useful in estimating, on the basis of general information, which aspects of water quality are most likely to be important in a given locality, and which parts of the aquatic ecosystem are probably most important in nutrient cycling, thus requiring the most intensive study. The basic information might, for example, be morphometry of the water body, its rate of exchange or flow, its transparency, and the nutrient content. Rawson and other (see O'Connor and Patten, 1968, Reservoir Fish. Resources Symp. p. 207) have found that plankton production of lakes can be somewhat predicted merely from morphometry. It is likely that this approach could be extended to rivers and estuaries. Williams (1968, Ches. Sci. 9: 42) has, for example, found that zooplankton abundance and importance in estuarine and coastal waters is primarily a function of mean depth.

Models which "sacrifice generality to (gain) realism and precision" are the type needed to analyze the structure and processes of particular aquatic systems, and presumably, to determine how a given sewage loading would affect the ecosystem and what modification of current practices would be required to achieve a particular level of water quality. While experience in constructing one such model might be of great assistance in constructing others, each such model will presumably be unique. No body of water is identical to others in terms of geography, ecology, and human usage. These detailed and comprehensive models will likely make use of many techniques, such as compartmental systems depicting the flow of toxic substances, nutrients, and energy, population models for important organisms, and complex statistically-based relationships evaluating the effect of one factor upon another. Watt (1968, Ecology & Resource Management, McGraw-Hill) has given many examples of "effects" models applied to ecological problems.

VALUE OR POTENTIAL VALUE OF MODELING IN EUTROPHICATION STUDIES

There are several different kinds of values that may result from a modeling approach. Most obvious is that models may provide the only way to assure that objectives for controlling the environment will actually be achieved if an adequate investment is made in various control alternatives. Likewise, through models it is possible to bring ideas and information together, thereby generating new knowledge.

The potential value of computer models for eutrophication has not been explored since there is little experience with these models. Because the eutrophication process involves many complex physical, chemical, and biological interactions which are time varying and which are stimulated by stochastic inputs of energy, nutrients, and the activities of man, the ultimate value of these models is too obscure to see clearly at this time.

The problems relating to eutrophication and other aspects of water quality are of such complexity that modeling appears to be the only feasible approach towards their solution. Not only are there the complexities of ecology and water and geochemistry to depict and interrelate but also the effects of human activities and the costs and side effects of proposed schemes for maintaining or improving water quality. The ability of models to clarify complex relationships is also important. The actual solution of water quality problems requires not only knowledge of what actions need to be taken, but also an informed legislature, judiciary, and general public to insist that proper action be taken. Models may provide one of the best means of conveying information to these groups. Knowledge held exclusively by the scientific community cannot solve public problems such as improvements of water quality.

A model of a eutrophication process cannot be considered entirely successful until it mimics the past, predicts the future, and serves as a base for system regulation. Clearly, this criterion will rarely be met. If one is dealing with low level nutrient pollution from fertilizer plant effluent, interest centers on ways of controlling input so as to maximize productivity with a minimum of esthetic upsets. This may require a manufacturer to reduce his output if more efficient processing means are not available. Alternatively, if one has a good understanding of the general influence of nutrient input on a seasonal basis, one may be in a position to require that the effluent be held and released during periods of the year when their effect is minimal. Reverse osmosis and precipitation techniques may be effective regulatory mechanisms in a limited number of cases but can only be regarded as supplementary for most large scale operations.

The usefulness of computers in a new application area is not likely to be appreciated at first. One reason for this is that a computer functions 10^7 times faster than humans. This is 7 orders of magnitude faster, which is almost impossible to perceive. For example, a jet plane flies at a speed that is only 2 orders of magnitude faster than humans walk. Seven orders of magnitude faster is almost equal to the speed of light.

Three specific kinds of values are usually derived from mathematical modeling. First, the exercise of constructing a mathematical model requires one to think carefully about his problem. This alone usually leads to new understanding of the problem, and the results to be produced by the model can sometimes be anticipated because the model was constructed.

A second value is that the rigorous structure of the mathematical model causes us to organize our data collection and other activities to support these models. The search for parameter values becomes directed, rather than amorphous, research. Needs for data not being collected may also be recognized. Other data may not be as important as originally thought, and collection of these may be terminated on the basis of insights gained from the model.

The third value is that mathematical models may be used to communicate ideas about a problem between people from different disciplines. It sometimes seems that the greatest value of many mathematical models is that they help explain to someone else what a problem is all about. This is especially important in water quality problems where different disciplines are involved.

PRACTICALITY OF IMPLEMENTING MODELS

Translating and Decoding (Parameter Relevance)

Every model involves coefficients for at least some of the terms. These coefficients are known as system parameters, and their numerical values are required for the model to represent system response. Since data are needed to quantify these parameters, they have certain desirable properties so that the data requirements for the model are minimized.

First, it is desirable that model response be insensitive to variations in model parameters. This means that inaccuracies in parameter estimates will not be reflected in the response. It also means that observations of system response will not lead to accurate parameter estimates, so indirect methods of parameter estimation are needed for these parameters. Second, parameters that do not change from system to system are desirable because they can be estimated once and used thereafter. Finally, parameters that change in a predictable way from one system to another are desirable because they can be evaluated for a few experimental conditions and then related to other parameters that characterize the system.

Generally a model is constructed to include important well-known relationships, and parameters are estimated from laboratory and/or field data. If one is fitting models to data for the sake of summarizing the data, a detailed evaluation and interpretation of the parameters is not of primary importance. On the other hand, if one is trying to justify selection of a particular model to explain experimental or field results, parameter considerations take on a new light. The more general the model (in the sense of its fitting observations from many different systems), the less useful it becomes in reconstructing the causative sequence of events that led to the observations. Frequently, examination of a rather primitive model will show that it has the necessary capability to simulate the real system by changing parameter values rather than by introducing additional variables.

A second point that is very important in discussing practical ecological problems is the reliability of parameter estimates. The errors associated with these estimates depend very strongly on the underlying structure of the statistical distributions involved. In a relatively simple model one can often predict what the error level will be; however, if the model is complicated by additional parameters (and for that matter other variables), it becomes virtually impossible to predict how large the error associated with the estimate of a given parameter will be. Certain models also will be very "ill-conditioned from a parametric point of view, which means that if estimation is an essential part of the modeling process, the model itself may have to be reformulated to provide a basis for realistic estimation. In the past a tremendous amount of effort has gone into the analysis of relatively simple systems, but until we reach a better understanding of the behavior of nonlinear models representing fairly complicated ecological processes, we will find that their usefulness is limited.

Measurement Problems (Data Gathering)

Since water quality models are tools for solving problems, such models will have little utility if they require information on quantities which cannot be measured at all or with the needed precision and accuracy. Microorganisms are, for example, highly important in the ecology of aquatic systems, and detailed information on the various groups of micro-organisms would presumably be quite helpful in constructing ecosystem models. It is, however, at present usually not possible to measure accurately either the number or biomass of microorganisms, or to separate phytoplankton respiration from that due to bacteria. There are, in fact, difficulties in measuring with precision either the biomass or physiological activity (or both) of almost every group of aquatic organisms. Thus, at the current level of knowledge of measurement of populations, models which do not demand much in the way of precise biological data may prove most useful. Precise physical and chemical data appears, in general, far easier to acquire.

Thus, one of the exceedingly difficult problems facing ecologists is prior system definition adequate to insure accumulating measurements on all relevant variables. This also means that sufficient insight must be brought to bear on the seasonal influence in the system so that sample timing is appropriate. Further, the sampling program must meet the need for good estimates of sample variation to be incorporated in subsequent analysis of model output, since simulated results will not be very useful until one is able to set reasonable bounds on their values. Unfortunately, financial considerations often limit sampling efforts to the detriment of sound statistical design.

TESTING THE VALIDITY OF AQUATIC ECOSYSTEM MODELS

A valid model may be defined as one that satisfies the goals for which it was built. Without goals, there cannot be a valid model.

The problem of testing the validity of a model is to see if the goals have been achieved. It seems desirable that some element of objectivity be included in the testing procedure. For example, comparison of predicted and observed system responses seems to be an objective test. However, choosing the validity criteria and the conditions whereby they are applied is a subjective step. Also, it may not always be possible to measure the system response quantity that the model has been built to represent. This is, in fact, almost always the case since most models are used to represent system response after the model has been developed and verified.

An important part of validity-testing should be to present arguments that the form of the model is adequate for the particular purpose for which it was constructed. For example, various terms may account for the forces that are believed to cause the system to respond in some particular way. The entire organization of the model may account for the overall structure of the system. When these arguments are presented, however, one should recall that every model is only a conceptualization of the system. In the end, the concept of model validity remains a subjective matter of judging how well the model objectives have been met.

ON MODELING THEORY

Modeling is an art more than it is a science. Part of the reason for this is the generally subjective character of the last mentioned step in the process, i.e. validity-testing. It is generally acknowledged among modelers that the greatest value of the process comes from the structure that models provide in a total research effort, and from insights which derive from the various alternative methods which may be used in implementation. The numerical output from a model rarely corresponds satisfactorily to empirical data. Hence, the real value of a model is not generally the nominal one which motivates modeling in the first place--mimicking system behavior.

There does not exist a formal theory of modeling. The modeling process as it is employed by engineers and practicing biologists is largely a cut-and-try, trial-and-error procedure. There is no rigorous formal structure which guarantees how output from a model will be related to behavior of the real system it is designed to represent. We are convinced that such a formal theory is possible, but clearly it is outside of our realm to attempt to provide it. What we have attempted to do in the paragraphs that follow, however, is to develop a sufficient formalism to identify the crux of the problem, i.e., to denote why models rarely correspond to "reality."

Consider two systems x and y. Let y represent the "real" (i.e. perceived) system, and x a model of that system. The two systems can be related by a binary relation M:

$$xMy$$

The relation M can be translated "is a model of." The properties of this relation are as follows.

First, x is a model of itself, and y is a model of itself also:

$$xMx$$

$$yMy$$

Thus, the relation M is reflexive. Secondly, the fact that x is a model of y does not imply that y is a model of x:

$$xMy \not\Longrightarrow yMx$$

The relation M is therefore asymmetric. As it turns out, this property of symmetry is the source of the difficulty in the modeling process. The fact that x is a model of y can also be written:

$$xMy \Longrightarrow yM^{-1}x$$

where M^{-1} is the inverse or transpose of relation M, translating "is modeled by."

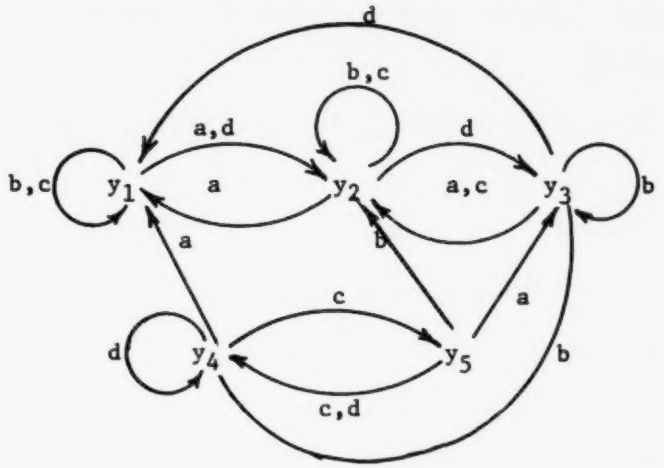
For a final property of M, we note that if x is a model of y and y is a model of z, then x is a model of z. Symbolically

$$xMy \text{ and } yMz \Longrightarrow xMz$$

The relation M is transitive.

Thus, we observe that the modeling process is one that is reflexive, asymmetric and transitive. Now let us examine the consequences of the asymmetry.

Consider the following system which we take to represent a "real" or "perceived" system:



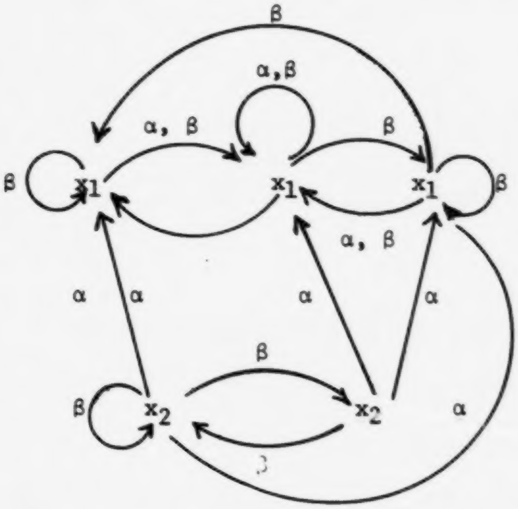
This diagram is intended to depict the behavior of a five-state system, the states being $y_1, y_2, \ldots, y_5$. The letters a, b, c, and d denote operators or transmittances which serve to transform the system deterministically from one state to another as indicated by the graph. The graph is called a behavior graph or kinematic graph. All possible behavior of this defined system is indicated.

Now suppose that we want to model this sytem. This means establishing a relation M between another system x, the model, such that this latter system mimics in some respect the performance of the system of interest. Many models are possible, but we select one for illustration.

A model M is a homomorphism between a "real" system and the model system. That is, a series of many-one correspondences between the two systems must be established to define M. One such set of correspondences follows:

x_1	y_1, y_2 or y_3
x_2	y_4 or y_5
α	a or b
β	c or d

Applying these correspondences to the original system graph above, we obtain:



This new behavior graph shows that x_1 transforms to x_1 if either transmittance α or β is applied. It also shows that x_2 transforms to x_1 under the influence of α, but transforms to itself under the influence of β. Thus, there is considerable redundancy in the graph immediately above, and the essential features of the new system are described by a simpler graph, as follows:

$$\alpha, \beta \quad x_1 \xleftarrow{\alpha} x_2 \quad \beta$$

This simplified graph is the behavior graph of the model system, and it is clear that it bears very little resemblance to the behavior graph of the original system. Furthermore, knowing the correspondences that were used to create the model M, we still cannot translate the model system back to the original system. To do this, it would be necessary to know the inverse correspondences, if they existed, that is we need to know M^{-1}. In general, this inverse relation is not known, nor do we have a theory of modeling which tells us how to find it, or even if it is knowable. Hence, our situation can be depicted as follows:

$$\begin{tikzcd} \boxed{y} \arrow[r, bend left, "M"] & \boxed{x} \arrow[l, dashed, bend left, "M^{-1}"] \end{tikzcd}$$

Having moved from the real system y to the model system x, the model system cannot be related back to the real system. This is a fundamental dilemma, it seems, and chances are that mathematicians have probably solved it but we simply do not know about it. We need to know (1) does M^{-1} exist and (2) if so, is it unique? Hence, existence and uniqueness, the twin problems of concern to all mathematicians, seem to be very significant in relation to the modeling process.

In practical terms, it seems that two options are available. First, since it is asymmetry of M which gives the problem in the first place, perhaps it may be possible to give attention to deriving models which are quasi-symmetric. Alternatively, it may be possible to develop empirical schemes which give adaptive or iterative approaches to estimating M^{-1}.

These may seem like esoteric and academic problems in the context of environments facing imminent irreversible damage, but we suggest that they will place critical restraints on modeling of the eutrophication process, or any other natural systems phenomenon, in the future. Therefore, we commend it to the immediate attention of mathematical specialists who may be in a position to help.

DISCUSSION

MODELS OF AQUATIC ECOSYSTEMS

PATTEN: To me the essential feature of modeling is that it is a measure of information. I agree that it is a subject to discuss. We have already established that X is a model of itself, but I doubt that you can establish this generally. If X is a model of Y, then Y is also a model of X.

CHEN: If you are modeling an ecosystem, for example, you can conceptualize a model.

BELLA: The obvious thing in the discussion is that you people are saying that the model obviously doesn't even approach the diversity of the real world. We know that diversity both in species and environment is very important in the real world. We can't possibly get this in our modeling, and so when we translate back to the real world, we are making judgments on a model that has very, very low diversity, and making these judgments for the real world in which diversity is very, very important. So there is a certain amount of judgment, and this is the question mark that you have.

PATTEN: This model represents a certain class of behavior available to the real system. You have a representation of a perceived system. You can look at it a hundred or a thousand different ways, depending on what you define the end relations to be. This model, this system, represents the full behavior of this system under these corresponding systems.

MORGAN: What is your general basis for selecting those models?

PATTEN: In this case it was arbitrary. Obviously the real case is the basis of all of our experience, all of our predictions, and the things that we want to know about the system. It goes back to that point. In order to model a system you must have an objective. You must define what aspect of that system it is that you wish to know about. Until you do that you cannot define your problem, you cannot define your objectives clearly.

MORGAN: I can't avoid thinking that some of the comments that I have heard in the last few minutes are not really directed to that issue, but they are rather directed to the issue of the lack of accurate detail of the structure of the real system. It doesn't seem to me that is the same thing as the end result.

PATTEN: Here we give a system that we know all about. Now the real systems, first of all, may have deterministic as well as stochastic components, and so on. The realities are much more messy. The point is, we can still take that messiness and translate it into something that we can handle pretty well. And we will do this with various degrees of satisfaction. We say to ourselves that we are going to prove this process. We learn the process of modeling that makes this diverse system come out with ideas. The class of model that we choose, and how we choose to implement it, also helps us to gain insight in this very general and selective thing about the real system. That's fine, but the point is that when we are done we turn the crank on the models and crank out the numbers on the theory, or however we wish to do it, then the problem is relating an output back to the input, and it seems to me that we are all pretty generally dissatisfied about this aspect of the modeling process.

ROBB: Your unhappiness with the modeling process, it seems to me, stems from the problem of trying to develop models and then see whether they will jive. The criterion for judging a model is whether or not it gives you the answer you want to receive from this particular model. This process, we all recognize, is a process of taking a shot in the dark and then asking people who are to use the model after it is developed whether it is accurate. The problem is in defining our objectives.

SHAAKE: I have mixed feelings about your comments, sir. I feel that it is important to be concerned, because this is what we are doing. I also agree that if all we do is think about it we are not getting it done, so we must take action and try to accomplish it. The whole business of setting the criteria is subtle, because you must include what is possible as well as what we would like to have so that it may represent part of the balance of equilibriums involved. I think that is true of the modeling. What we think are good models because they satisfy certain criteria might possibly lead to a bad model.

BREZONIK: I think there is a problem, and it is this: the problem depends on the objectives of your modeling. If you have an objective of meeting some water quality standards, for example, or getting some applications, any model that you make that meets these objectives is fine, whether it is a realistic model of the real world or not. A stochastic model, or an analytical model, or the simplest model that you can devise, if it works to meet those applied goals, that's a good model. But this is not true if your objective is to study the real world of causal relationships and get an understanding of the basic processes of all three models. In that case, then you do have your problems. How does your model relate to the real world?

THE PROCESS OF EUTROPHICATION AND CRITERIA FOR TROPHIC STATE DETERMINATION

A panel position paper

by

Jack R. Vallentyne

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THE PROCESS OF EUTROPHICATION

The classification of "harmonic" lakes in the oligotrophic-mesotrophic-eutrophic series originated with Naumann in 1919 and has since been developed and used by numerous investigators. The concept was originally developed for the classification of lakes per se in relation to natural eutrophication. In attempting to apply the trophic scale to modern problems of cultural eutrophication a number of difficulties arise:

1. Investigators in different regions have divergent concepts of the oligotrophic-eutrophic scale. An oligotrophic lake in southern Canada, for example, might be classified as mesotrophic if in Norway.

2. Natural eutrophication is a slow process in relation to cultural eutrophication. Although the results in both cases are similar in many ways, the former in a given lake is largely a function of morphometric change, whereas the latter occurs without significant change in morphometry and is largely determined by increases in nutrient load.

3. There has been a confusion in the limnological literature between definitions of trophic state based on causes (e.g. nutrient supply, nutrient concentrations, climate, etc.) and effects (e.g. abundance, production and composition of planktonic algae, benthic vegetation, bacteria, benthic animals, fish, etc.).

4. With the present need for control of cultural eutrophication and models to facilitate an understanding of control mechanisms, there is a priority on measurements in c.g.s. units so that effects can be linked directly to causes and expressed as functions of causes. Indicator species or associations, though extremely useful, do not fulfill this requirement.

In discussing the above one member of the panel (Shapiro) made a case for the development of a eutrophication scale based on several criteria (e.g. Secchi depth, species, algal abundance, etc.), comparable in some ways to the Richter scale used in seismology. After much discussion it was generally agreed that the "Shapiro scale" would be useful in providing the public with a meaningful guide-line, but it would not be useful in modeling because of the confusion that would result from the addition of "apples and oranges".

The panel and audience, after much heart-rending discussion including diverse arguments of the "But that does not account for" type, came to the following general conclusions:

1. The concept of trophic state as described in the limnological literature (e.g. oligotrophy, mesotrophy, eutrophy) is too multi-dimensional and too susceptible to individual interpretation to be of value to model makers. Nevertheless, limnologists should establish criteria for oligotrophic, mesotrophic, and eutrophic waters, both in terms of lake classification _per se_ and problem-oriented classifications independent on morphometric change. Both have predictive value.

2. Criteria for the assessment of lakes in relation to human use must be based on the biological effects that constitute the problems, rather than on chemical and other causes that accelerate or initiate biological processes and changes. The reason for this is very simply that no one has as yet been able to quantitatively relate causes to effects for lakes in general. Thus, while nutrient loading rates and the relative percentages of natural and man-derived nutrients provide the best available criteria for control at the present time, they do not permit any assessment of effects on water use.

3. The panel agreed that with regard to biological effects in lakes the most useful measurements at the present time are as follows:

(1) Standing crop of algae in units of chlorophyll or chlorophyll a (mg/m^3) and in units of cell volume (cm^3/m^3). These measurements should refer only to the euphotic zone with checks made occasionally to make sure in the case of chlorophyll that the proportion of phaeophytins and pheophorbides is low.

(2) Primary production of algae in units of mg C/l.day both as a productive season average and during blooms. Primary production in units of mg C/m^2.day was considered of much lower significance in relation to human use of water. This is because of the tendency for clear waters with low phytoplankton concentrations and few eutrophication problems to yield relatively high production values when integrated over depth. More attention should be given to assessment of primary production by benthic algae.

(3) Secchi depth in meters in cases where turbidity due to mineral matter or low light penetration as a result of dissolved humic materials do not constitute major interferences. In this sense Secchi depth is a rough index of algal biomass in the upper water layers. It is a simple and useful measure that can be applied by unskilled personnel.

Guide-lines demarcating acceptable and dangerous levels (Productive seasons) of the above are: (1) 5-10 mg chl a/l, 3-5 cm^3/m^3; (2) 0.20-0.4 mg C/l.day; (3) 3-6 m listed in Table 1.

Some weighting for lake morphometry should be taken into consideration in using these guide-lines. In very large lakes, for example, overall water usage decreases exponentially from the shore-line to the geographic center. In such cases lake-average data may be less meaningful than along the waterfront or at beaches.

A variety of other indicators are available for assessment of parameters of "trophic state", but for one reason or another these are not so useful as the above in relation to modeling. Among these are:

1. Oxygen depletion and oxygen depletion rates in the hypolimnion. In general when the average dissolved oxygen level in the lower half (by volume) of the hypolimnion is less than 3-5 mg O_2/l during conditions of restricted circulation, repercussions to cold-water fish populations can be expected even though the fish may have a refuge in more highly oxygenated waters of the upper part of the hypolimnion. In bodies of water too shallow for a hypolimnion, pronounced oxygen depletion can occur in bottom layers during prolonged periods of calm weather at the sediment-water interface - even though those waters are initially near saturation.

2. Sawyer's rule that algal problems can be expected whenever the concentrations of inorganic-N (ammonia, nitrite, nitrate) and inorganic-P (orthophosphate) exceed 300 μ g-N/l and 10-15 μ g P/l at the time of spring turnover.

3. Shape of primary production vs. depth curves. Curves without pronounced maxima in the upper layers are generally characteristic of clear waters without algal problems, whereas those with maxima in the upper layers tend to be associated with algal problems.

4. A variety of indicator species and species associations are available which can delineate stages in eutrophication. Thus, waters with algal problems generally have high blue-green algal populations (e.g. _Anabaena_ spp., _Anacystis_ spp., _Aphanizomenon_ spp.). Also, some diatoms are indicative

of early stages in eutrophication, increasing in frequency prior to major deleterious changes. Comparable indicators of "trophic state" exist among benthic plants (Cladophora) and animals (oligochaetes, midges). These can be most useful in general assessment of pollutional levels, but have the disadvantage in relation to model studies that the data cannot at present be given in cause-and-effect terms, nor expressed in c.g.s. units.

The panel believed that more attention should be devoted to developing eutrophication criteria based on sediments and sedimentation rates (P, N, well-preserved organisms or their parts, etc.) since they offer a time-dampened and space-dampened source of information. Also, recent trends can often be established from near-surface sediment cores, i.e. information relating past to present conditions.

Finally, the panel stressed that the assessment of "trophic state" and the interpretation of the parameters cited above requires much biological insight, particularly in rapidly changing situations. In this sense, interpretation and assessment demands the expertise of a broadly diversified limnologist familiar with the analysis of biological communities at the species level.

DISCUSSION

THE PROCESS OF EUTROPHICATION AND CRITERIA FOR TROPHIC STATE DETERMINATION

SHAPIRO: We are not trying to ascertain the trophic state to determine whether or not we do anything about the lake. That is another matter. What creates that trophic state is really secondary to giving it a name or a number. No matter what the morphology of the lake is, it is what it is, and that is what we are trying to describe. We are trying to describe it so that we can say that at this moment in time it is such and such. And whether it is that, may be because it is a shallow lake. If we want to establish the trophic state in the context of doing something about the lake, then I think you also have to go back and find out: What was that lake like before it was disturbed? Therefore, we have to find out how far back we hope to get, what the ultimate limit would be to recover this lake, and this opens up a whole new conception of things. The potential trophic state. Maybe we can define the trophic state as a point in time.

VALLENTYNE: What limnologists were doing years ago was classifying lakes just to be classifying lakes, clarifying in their minds their purposes, against the interest and importance that attaches to the subject today.

SHAPIRO: You want to assess the present status of this lake for this year. You want to know how it has changed from last year. To do this we must come up with some objective statement. We must have some way of comparing this lake with another. We must be able to say, this lake is better than that one, and this one is a little worse than that one. If there are enough gradations on the scale, it doesn't matter whether it is 22nd or 29th. It is a way of finding out where you are in the ball park.

SHANNON: I agree with Dr. Shapiro, we should have one number, I think one of the disadvantages in considering all of these different criteria is that we get different results. If we properly weigh the criteria we can compensate with give and take. This is the sort of situation where we can give a little in one criteria, and so on, until we get one number that will prevent some sort of conflict in the various criteria.

WATTS: How does this explain numbers to me?

SHAPIRO: How does a seismologist explain an earthquake to you? He tells you the earthquake is monitored on a scale of ten. You see it somewhere in limbo. If you hear that there has been an earthquake somewhere in terms of the Richter Scale you just know there has been an earthquake. You don't have to model it, you know it.

VALLENTYNE: What you are talking about here is the value of such a scale in explaining the situation to the public.

SCHAAKE: There must be some measure that needs to be explored to decide whether certain kinds of lakes are desirable or undesirable, for one reason or another, and that needs to be done as part of this, perhaps some scale of measure, something that has a scalar measure to it of some sort. If you want to talk about lakes in terms of their desirability, one should investigate what about a lake makes it desirable or undesirable. If it is a scientific value, then I think that needs to be considered. It takes a combination. You have to validate something.

BEETON: I think that limnologists simply use the terms eutrophic, mesotrophic - I think we have an obligation at this point to define these a little bit more precisely. We find that they are being used now by many people. Whether or not you want to throw these out, that's all beside the point. You still have this obligation to define them. You have to find criteria for at least these categories, if we don't do anything more. But as far as going to some compound index or something, this might be useful in terms of establishing waterflow criteria, or something like that, and as I stated earlier, I think we have got to follow two things, and one of them is the practical application. The other is, what we would want as a scientific community. I think, as a limnologist I wouldn't be unskeptical. I wouldn't want to accept somebody's index for a lake, because I would like to know just what he found and what he based this on.

SHAPIRO: You are finding fault with something that doesn't exist yet. This could be moderately sophisticated. For example: You could have a method of determining whether or not a lake has phosphate. Do the algae have alkaline phosphatase? This sort of thing could be run into it. This could be a metabolic analysis of the health of the lake. There needn't be something where people could make a great big mistake. I think we've got to come up with some objectivity in this.

FOX: As far as a way of communication between scientists by using this, not as an index number, but as a code like a ten digit number which would give us a range of all of the parameters of a lake.

VALLENTYNE: That would be useful. This might be good as an index to scientists who weren't particularly involved in the maintenance of records. For the ones who did, it could be rather confusing.

SHAPIRO: We need a code. A code could embody an awful lot of information. Some of these things could be fairly simple, for example: In this particular lake you may be using the rate of oxygen depletion, or whether the oxygen disappeared the first, second, or third month after stratification or something.

BARTSCH: I think we've perhaps exhausted this aspect of things. Where some of this approaches communication with the public, I think the public is becoming aware that there is a word, eutrophication. I think the public is approaching the point where we need something more than simply saying that this lake is eutrophic. We may need advice to see how the public is better informed. We need to know something more than that this lake is eutrophic. If we begin to classify lakes in terms of the system, or on the semi-Richter scale, or whatever you want to call it, I think the second point is that we seem to be confusing the point with the question of whether this is the best way of communicating among limnologists, or among other scientific people who are concerned with eutrophication. I think it helps that a limnologist, or the guy who is interested in lakes, or in eutrophication, is not going to take the number, however it is constituted, and say, I accept this and I am going to use it from here on. He is going to go back and look at the basic information upon which this number was derived. I think the third point is whether or not we are really talking about this in terms of the basic theme of this session, which is, How do we use this, or how can we use it, or what is the nature of the information we need in order to feed it into the modeling process?

BREZONIK: I agree that one number to classify a lake, say to classify lakes on a scale from one to ten, would not be too meaningful, but I think if you take three things: and one of them would be the chemical nature of the water, another would be climatic and morphometric properties, the third would be productivity, type of organisms, perhaps something like this would be meaningful as a short term handle on the lake.

BEETON: Criteria for classifying lakes are: mean depth; transparency; average dissolved oxygen in bottom waters during the late summer; the total dissolved solids. For some reason this seems to relate quite well to fish productivity in British Columbia. We can present another criteria: the average dry weight of plankton; the average dry weight of benthos; and then fish production. I looked at a number of criteria of what people thought were acceptable levels for classifying lakes in terms of eutrophic, non-eutrophic and mesotrophic, and tried to classify the Great Lakes in terms of these. I included the same things that Rawson had done, but I also included total phosphorus; silica, sulfate and conductivity; and then the species that were dominant in the benthic growths, as well as the species that were dominant in the phytoplankton community and the fish community. These are the criteria that have been used in many environments throughout the world, and people have published on these.

VALLENTYNE: We need to develop an index, not so much for the benefit of scientists and limnologists, as for the general public.

EUTROPHICATION: THE PROCESS AND ITS MODELING POTENTIAL

A Position Paper

by

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INTRODUCTION

Concern over excessive fertilization of lakes has been expressed for many years, at least with regard to certain lakes. For example, nutrient problems of the Madison (Wisconsin) lakes have been discussed for over fifty years (see Sarles, 1961, for a review). On a national scale, cultural eutrophication has been recognized as a serious water quality problem at least during the last decade, and a major effort to achieve a solution has been put forth by federal and state governments and by university researchers. Considering the importance of the issue and the comparatively long time during which it has been observed and studied, it is both surprising and disconcerting to note how little is known about eutrophication, particularly on a quantitative basis. Progress in obtaining solutions has been impeded, at least in part, because the process, its causes and its effects are vaguely defined and often confused. Only in the most general sense is there any agreement about the nature of eutrophication. What is excessive fertilization to one worker may be insufficient fertilization to another; one limnologist may classify a lake eutrophic; another might not. In essence, there is presently little or no agreement on the quantitative aspects of eutrophication. Before the problem can be solved, before mathematical models of eutrophication can be constructed, this deficiency will have to be remedied.

THE DEFINITION OF EUTROPHICATION

Eutrophication is a complicated chemical and biological phenomenon with ramifications extending to political and socioeconomic considerations. The phenomenon itself (aside from any spillover effects) can be divided into two parts. First, the process of eutrophication is simply the nutrient enrichment of natural waters without reference to the nature of the enrichment process. Nitrogen and phosphorus are usually considered the main eutrophying elements, but trace metals and organic growth factors may also be important. The process is described only in qualitative terms; e.g., quantitative nutrient loading rates do not define specific eutrophication rates. The second aspect of the problem is the effect of eutrophication on the water quality (or trophic state) of lakes. Eutrophy (the state resulting from eutrophication) is a lake state defined by various chemical and biological conditions; it is a hybrid concept. There are presently no quantitative units to express eutrophy. Consequently, it is presently not possible to relate the process of nutrient enrichment to its effects on trophic state in a quantiable manner.

THE PROCESS OF EUTROPHICATION

Eutrophication is not a new phenomenon, nor is identification of the problem particularly recent. For example, Cowgill and Hutchinson (1964) concluded from sediment core analyses that cultural eutrophication occurred in an Italian lake during the time of the Roman Empire. The precise origin of the term "eutrophication" is somewhat obscure; Hutchinson (1969) cites Weber (1907) as introducing the German equivalents to the terms oligotrophic, mesotrophic and eutrophic, to describe soil solutions in German bogs, but Naumann (1919) is generally credited with introduction of the terms to limnology. Naumann used the term "eutrophic formation" to describe a phytoplankton assemblage in nutrient rich waters. Similarly, an "oligotrophic formation" referred to the phytoplankton assemblage in nutrient poor waters. Later Naumann (1931) defined eutrophication as "an increase of the nutritional standards (of a body of water) especially with respect to nitrogen and phosphorus." In a classic paper, Hasler (1947) considered eutrophication as "enrichment of water, be it intentional or unintentional." Enrichment was broadly interpreted to include all nutritive substances.

Hasler (1947) and earlier, Strom (1928, 1930) and Lindeman (1942) felt that all lakes are initially oligotrophic. They regarded eutrophication as an inevitable successional process in lake aging and extinction, and the eutrophic state as a natural stage following initial oligotrophy and intermediate mesotrophy. Lindeman (1942) and Hutchinson and Wollack (1940) felt that the initial period of oligotrophy was relatively short. Hasler's (1947) concept of eutrophication as a consequence of lake aging is shown in Figure 1. He viewed eutrophication as a comparatively rapid, autocatalytic (i.e. self-accelerating) process. Hasler cited Mortimer's (1941, 1942) proposal that acceleration results from larger releases of nutrients from the reducing muds of eutrophic lakes than from the oxidized sediment layer of oligotrophic lakes. According to Hasler, artificial or domestic fertilization results in almost instantaneous eutrophication. In both Lindeman's (1942) model (Figure 2) and Hasler's (1947) model (Figure 1) eutrophication eventually levels off, and a long period of relatively constant production ensues in the lake. This stable period is termed the stage-equilibrium, during which the sediments are said to act as a nutrient reservoir or trophic buffer to maintain high production. During the stage-equilibrium, sediments continue to accumulate, and the lake approaches extinction. Hasler's model suggests increased production as the lake approaches extinction, but he did not describe the final successional stages in detail. Lindeman proposed a period of senescence when the lake becomes too shallow for maximum phytoplankton growth. During senescence the lake is characterized by a large littoral area predominated by rooted aquatics and by increased marginal vegetation. Production per unit volume of lake water may reach a maximum during these times, but according to Lindeman's model production per unit area is lower. Eventually the lake becomes a marsh or bog and fills in completely to begin terrestrial stages of succession.

Recent studies have questioned the classical scheme of eutrophy inevitably succeeding oligotrophy. Mackereth (1965) theorized that lakes may be more productive in their earlier stages (shortly after glaciation in this case) than later. On the basis of sediment core analyses in the English lake district Mackereth proposed that lakes typically proceed from a eutrophic origin toward an oligotrophic state unless another natural or human disturbance alters their course. The concept of glacial scour causing initial eutrophy is not new; for example, Lindeman (1942) and Hutchinson and Wollack (1940) recognized that the influx of phosphorus could be high shortly after glaciation in some geological regions. Brundin (1958) found glacial erosion to be responsible for eutrophy in Lake Juvvatnet, Sweden. This is a true glacier lake (i.e. the base of a glacier forms part of its shore), and Brundin concluded that the glacier ice is a rich source of phosphorus in the form of glacier-ground mineral particles of colloidal size. Livingstone and Boykin (1962) reported phosphorus distributions in a sediment core from Linsley Pond (Connecticut) which can be interpreted according to Mackereth's hypothesis of eutrophy preceding oligotrophy. The ontogeny of Linsley Pond's trophic state is still unsettled (e.g. Brooks and Deevey, 1963; Cowgill, 1968), but the above studies indicate that the classical concept of succession from oligotrophy to eutrophy may not be universal.

Ecological theory also supports a natural tendency for lakes to progress from eutrophy to oligotrophy. The eutrophic state has many characteristics of immature ecosystems (e.g. low stability and low species diversity) while the reverse is true of the oligotrophic state (Odum, 1969). Thus a lake should tend toward the oligotrophic state if left alone. In this sense, eutrophication can be viewed as resulting from a disturbing influence (a forcing function) from outside the lake (i.e. natural or cultural nutrient enrichment rather than from a natural internal tendency.

The Hasler-Lindeman concept of natural eutrophication clearly considered the process to be an irreversible succession in a lake's evolution - or more precisely in its devolution. Hasler (1947) apparently considered cultural eutrophication in the same light and made no distinction between the two processes with regard to the lake's subsequent ontogeny. This view was generally accepted until quite recently although Hasler himself now views cultural eutrophication as reversible (Hasler, 1969). Cowgill and Hutchinson (1964) and Hutchinson (1969) questioned the supposed irreversibility of eutrophication and reported a case of apparently reversible eutrophication during the time of the Roman Empire. Lago de Monterosi rapidly became eutrophic when the Romans exposed a nutrient richer limestone stratium by cutting trees and constructing a road by the lake around 171 B.C. Later (in postimperial times) the lake became less eutrophic. Association of cultural eutrophication with inevitable lake aging has spawned some unfortunate and misleading comparisons in the popular press, e.g. equating eutrophic lakes with "dead or dying lakes." The irony of the comparison is that eutrophy implies an abundance of life, which results from high nutrient conditions.

As a result of recent sediment core studies, some have questioned whether natural eutrophication (in the classical sense) actually occurs. One can only speculate, but it is probably not a universal feature of lake aging. Perhaps it is even the exception rather than the rule. It seems apparent that the life history of lakes is considerably more complex than Hasler's and Lindeman's models imply. Undoubtedly trophic state changes with time, and young deep lakes, may often appear oligotrophic compared to their older shallow counterparts. This would more likely result from changes in physical characteristics of the lake itself than from changes in nutrient inputs. Of course, if a lake's nutrient content and productivity increase, the lake is more eutrophic, no matter what the nature of the enrichment process is. There is little reason to expect a sudden or marked increase in a lake's natural nutrient budget at some time long after its basin has been formed. On the contrary, a geologically recent basin should have a greater abundance of leachable nutrients than a mature landscape. Sudden increases in natural nutrient influx could result from catastrophic events such as earthquakes or volcanoes but would not seem to be a general feature of the terrestrial erosion cycle from youth to old age.

If some lakes begin as oligotrophic and later become eutrophic, or vice versa, it is also probably true that many lakes remain oligotrophic or eutrophic throughout their entire life span. According to Eddy (1963), soft-water lakes in northeastern Minnesota are typically oligotrophic, reflecting the geochemistry of the region (nutrient poor Precambrian bedrock covered by a thin layer of red drift). Lakes on the Canadian Shield of western Ontario are oligotrophic for similar reasons. In north-central Florida, solution lakes in the nutrient impoverished, sandy soils of the Trail Ridge are also oligotrophic and will probably remain so unless disturbed by man. These lakes could age directly into the senescent state without ever being demonstrably eutrophic.

Although some eutrophication may be desirable in certain lakes to increase fish production, in general the process implies a loss in water quality. Whether natural eutrophication causes the same type and degree of degradation as cultural eutrophication is questionable. It would seem that cultural eutrophication has potentially more severe effects. There probably are limits for rates of nutrient enrichment from natural sources, which are only rarely exceeded. By comparison cultural nutrient enrichment has few geochemical constraints; it is furthermore superimposed on the existing natural nutrient sources. It would also seem that natural eutrophication is less likely to upset the ecological balance of a lake.

On a geological time scale, cultural eutrophication is almost instantaneous. An affluent society can change an oligotrophic lake into a highly eutrophic one in a few decades or even a few years. This step function change may cause undesirable transient responses which probably do not occur in a slower natural change.

FACTORS AFFECTING EUTROPHICATION

Nitrogen and phosphorus have most often been cited as the main eutrophying elements, but there is little agreement among authorities concerning their relative importance as limiting elements. Arguments can be made for both and algal bioassays have found each limiting primary production in different lakes. Many other elements and compounds (e.g. trace metals and vitamins) are essential for algal growth. Recently, CO_2 has been implicated as exerting a major role in the stimulation of blue-green algal blooms (Lange 1967; Kuentzel, 1969). However, comparatively little is known about the roles of these substances in eutrophication. There are reasons - based on geochemical and biological considerations - to believe that lakes generally are limited by their nitrogen and phosphorus inputs, but unambiguous proof would be nearly impossible. It is likely that inputs of minor essential nutrients are correlated with one or both of the major limiting elements.

Factors affecting nutrient enrichment of lakes can be classed as either natural or cultural phenomena (Table 1). The natural nutrient load imposed on a lake is a function of drainage basin geochemistry and hydrology, and climatic conditions. Superimposed on these are a variety of human or cultural elements, e.g. urban and agricultural runoff and the amount of domestic sewage disposed into the lake. Furthermore, human factors can affect some of the natural forces. For example, mining operations may change the geochemistry and hydrology of the basin, and air pollution may increase the nutrient content of rainfall. For a given total nutrient influx, net enrichment may vary depending on the temporal variations in the input (whether it is a periodic slug or steady addition).

The effects of nutrient enrichment of a lake's trophic state are controlled by numerous physical and chemical factors (Table 2). Rawson

TABLE 1

Factors Affecting Nutrient Enrichment Rates (Eutrophication) Of Lakes

Natural Factors

- Geochemistry of the basin
  - (Composition of underlying rock structures)

- Soil types
- Hydrology
  - Size of drainage basin
  - Short-circuiting
  - Detention time in lake
  - Groundwater composition

- Climate
  - Precipitation
  - Thermal structure

Human Factors

- Domestic sewage
- Agricultural runoff
  - Type of farming
  - Fertilization practices and extent

- Mining operations
- Industrial wastes
- Urban runoff
  - (Auto exhaust, lawn and garden fertilizing, leaves, etc.)

- Nutrient leaching from drained marshes and from garbage dumps

TABLE 2

Physical and Chemical Factors Controlling The Effects of Nutrient Enrichment on Trophic Status

Physical	Chemical
Mean depth	pH
Steepness of bottom contour	Balance of all nutrients needed for production
Shoreline irregularity	Suspended solids (as affecting transparency)
Percent littoral area	Nutrient concentrations
Mean depth/surface area ratio	Dissolved oxygen
Wind protection by surrounding terrain	
Temperature	
Insolation	
Circulation which affects sedimentation rates	

(1939) organized the environmental factors affecting lake production in the classical diagram shown as Figure 3. Limnologists have long realized that lake productivity is influenced by factors other than the concentrations of nutrients. Other factors influence lake productivity primarily by affecting the distribution, availability, and utilization of nutrients; i.e. they influence productivity only indirectly. Consequently it is difficult to assess their individual importance. Similar statements can be made regarding the effects of these factors on a lake's general trophic state.

Morphometry markedly influences the availability and distribution of nutrients in a lake. Important morphometric parameters in this regard include mean depth, steepness of bottom contour, percent littoral area, shoreline irregularity and mean depth/surface area ratio. Rawson (1955 and earlier papers) considered mean depth of fundamental importance in lake productivity. He found a hyperbolic relation between mean depth and long-term fish production in the five Great Lakes and seven large western Canadian lakes, indicating a rapid increase in fish production as mean depth decreased below twenty-five meters and a slow decrease in production as mean depth increased beyond twenty-five meters. Similar results were obtained relating phytoplankton and bottom fauna standing crop to lake mean depth, and Rawson suggested that a mean depth of 20-25 meters is the dividing point between oligotrophic and eutrophic lakes. The effect of mean depth on lake productivity is probably explained largely because nutrient replenishment from the underlying sediments in shallow unstratified lakes is more likely than in deep stratified lakes.

Climatic factors, including temperature, insolation, and wind (as it affects circulation patterns in the lake), would seem to affect all lakes within a geographic region to similar degrees, but climatic differences may arise from differences in lake basin morphology and surrounding geography. For example, two lakes in the same region are likely to be affected by wind to different degrees if one is surrounded by high forests and the other by pasture land, even though a uniform wind pattern may exist in the region. Between geographical regions with pronounced climatic differences, factors like temperature and seasonality may markedly influence the eutrophication process and its manifestations.

Availability of nutrients is also affected by the chemical composition of the water. A proper balance of all nutrients is needed for optimum production. Supra- and sub-optimal concentrations of various ions occur and affect productivity and the nature of the dominant organisms in various ways. The presence of certain microfactors may partially obviate the requirements for other nutrients. Some organic factors may not be necessary in an absolute sense, but their presence may eliminate the need for their synthesis by growing algae. Vitamins are representative of such factors (although many algae have an absolute requirement for some vitamins). The ionic forms of many nutrients are functions of pH, and their transport into cells may be pH-dependent. The possibility for CO_2 limitation of photosynthesis at high pH is such an example. Also, cell membrane permeability is pH-dependent. Organic color may influence production by affecting pH, by chelating certain trace metal nutrients and by absorbing light. Suspended solids can affect production by reducing light transparency and by sorbing essential nutrients. Dissolved oxygen affects availability of nutrients, especially in relation to sediment regeneration and leaching. It is well known that nutrient release from sediments is enhanced by anoxic conditions in the overlying water. Numerous other examples can be cited, but the above discussion clearly indicates that a multiplicity of physical and chemical factors control the availability of nutrients in natural waters.

LAKE TROPHIC CLASSIFICATION

As originally defined, the terms oligotrophic and eutrophic referred to water types that are defined (ideally) by chemical composition. Use of these terms later became less precise, and they were also taken to denote lake types in which edaphic and morphometric factors affect the water's chemical and biological characteristics. Among present-day workers there is little or no distinction between these meanings; probably the terms are used more frequently to denote lake types than water types. Hutchinson (1967) criticized this extension of the terms and proposed a return to the original definitions. However, the present usage is probably too widespread for retrenchment, and if the original definitions were reemployed the problem of naming lake types would still remain. Also, the chemical and biological composition of lake water is at least partly a product of edaphic and morphometric circumstances.

As mentioned earlier, trophic states are defined by numerous physical, chemical, and biological criteria. The problems and limitations of trophic criteria are discussed by another panel. Table 3 summarizes the common trophic indicators and the manner in which their values differ in oligotrophic and eutrophic lakes.

Lakes are too diverse to fit meaningfully into only two classes. Aside from classification of lakes according trophic levels, numerous other classifications have been devised based on one or more of the following:

TABLE 3

Trophic Criteria and Their Values In Oligotrophic and Eutrophic Lakes[1]

Physical	Chemical	Biological[2]
Transparency (D) (Secchi disc reading)	Nutrient concentrations (I) (e.g. at spring maximum)	Algal bloom frequency (I)
Morphometry (D) (mean depth)	Chlorophyll *a* (I)	Algal species diversity (D)
	Conductivity (I)	Littoral vegetation (I)
	Dissolved Solids (I)	Zooplankton (I)
	Hypolimnetic oxygen deficit (I)	Fish (I)
	Epilimnetic oxygen supersaturation (I)	Bottom fauna (I)
	Sediment type	Bottom fauna diversity (D)
		Primary production (I)

[1] (I) after criteria signifies values increase with eutrophication; (D) signifies values decrease with eutrophication.

[2] Biological criteria all have important qualitative changes, i.e. species changes as well as quantitative (biomass) changes as eutrophication proceeds.

lake morphometry, thermal stratification patterns, geography and climate, sediments, chemical characteristics, flora, etc. With respect to trophic classification, it has long been apparent that many lakes readily fit into neither the oligotrophic nor the eutrophic class. Various other classes have been proposed by limnologists to accomodate these lakes and to facilitate more precise classification. For example, the mesotrophic class was originally introduced by Weber (1907) to be an intermediate class between oligo- and eutrophy. The terms hyper-eutrophic and ultra-oligotrophic have been proposed more recently to represent the extremes of the two main classes, but their characteristics have not been accurately defined. Precisely at what point a lake changes from eutrophic to hyper-eutrophic is somewhat subjective. It should be noted that neither ultra-oligotrophy are universally recognized as distinct states. However, few would question the utility of having more than two terms (oligotrophic and eutrophic) to classify lake trophic conditions.

The senescent state was introduced by Lindeman (1942) as the final lacustrine successional phase. This trophic state is characterized by shallow lake basins, relatively large littoral zones, large standing crop of macrophytes and an abundance of nutrients (which may be largely bound in the standing crop). Senescence is not universally recognized as a distinct trophic state and has received relatively little attention in the literature. The senescent class has special importance in Florida, where many lakes are less than ten feet deep and border on the senescence state if not indeed senescent.

The role dystrophy in lake typology and succession has been the subject of much controversy (e.g. Berg and Peterson, 1956; Jarnefelt, 1952, 1958).

The term "dystrophic" (literally, ill-fed) generally refers to lakes with high color and organic matter and low calcium and pH. Such lakes may have high or low nutrient conditions although the latter are perhaps more common. Dystrophic lakes usually have low productivity and a limited flora and fauna, but these facts reflect the limiting conditions imposed by environmental factors other than nutrient levels (e.g. low light penetration and low pH caused by organic color). Many Florida lakes receive swamp drainage and have high organic color as well as other characteristics of dystrophy. Dystrophic lakes are common in forested lake regions (e.g. northern Wisconsin, Maine, Scandanavia).

Naumann's original classification was ambiguous regarding dystrophy, but Thienemann (1921, 1925) divided lakes into the following classes:

I. Lakes with colorless water
    a. oligotrophic lakes
    b. eutrophic lakes
II. Lakes with brown water

Because colored lakes were not well studied, Thienemann did not subdivide them. Hansen (1962), states that Thienemann clearly intended a bipartite or quartered system based primarily on color and subdivided according to nutritional or trophic bases, but that his system has usually been misinterpreted (e.g. Naumann, 1931) as tripartite (i.e. three principal types: oligotrophic, eutrophic, and dystrophic lakes). Many American limnologists still interpret dystrophy as a lake type parallel to oligotrophy and eutrophy although the former has no intrinsic relation to nutrient content which in essence is the basis for trophic classification of lakes.

Because the distinguishing characteristics of dystrophic lakes are related to organic color rather than nutrient concentrations, dystrophy is a poorly chosen term. Hutchinson (1967) proposed adoption of Järnefelt's (1953) almost unpronounceable term "chthoniotrophic" in place of dystrophic. This may be more precise etymologically but is certainly less convenient and aesthetic. It still suffers from the connotation of nutritional relationship because of its "trophic" suffix. Jarnefelt himself did not use the term in a later paper on lake typology. The term "tyrfotrophic" has also been used by a few to denote dystrophy. Hansen (1962) proposed a refined version of Thienemann's original bipartite system. He classified lakes as (1) colorless or gyttja lakes and (2) dystrophic (colored) or dy lakes. The "gyttja" and "dy" terminology derives from the sediment types found in each class. Oligotrophic and eutrophic subclasses occur in each category. Use of the terms "oligohumous" and "polyhumous" (euhumous) to denote classes 1 and 2 respectively was rejected by Hansen because of ambiguities associated with the term humus in soil chemistry. However, Stewart and Rohlich (1967) used these terms, which do offer some descriptive advantages, as long as it is realized that the nature of organic color in water may not be humic in the original and specific sense in which the word is defined for soil fractions. Colorless and dystrophic lakes may be fundamentally different and may develop and age differently; the nature of eutrophication in the two classes of lakes may be dissimilar, and different criteria or criteria values may be required to define eutrophy in oligo- and polyhumous lakes. The dystrophic classification has been further discussed by Brezonik _et. al._ (1969)

The above discussion indicates the need for more precise definitions and delineations of lacustrine trophic states. The most difficult problem rests in the setting of boundary conditions, i.e. where should the line be drawn to separate, for example, mesotrophic and eutrophic lakes. Whatever the position of natural eutrophication in lake evolution - and it may have several positions depending on a lake's individual circumstances - it seems proper to distinguish natural and cultural eutrophication as clearly separate (non-identical) processes. Whether it will also be necessary to distinguish cultural eutrophy and natural eutrophy as distinct states cannot yet be stated. It may be instructive to recognize Lindeman's senescent stage as another major lake class. Dystrophic lake classification problems still remain; the bipartite systems described by Thienemann and Hansen seem logical, but arguments over nomenclature, particularly for the trophic subclasses of colored lakes, need resolution.

On the other hand, it is quite possible that lakes are not meant to be classified so simply and unequivocally. The wide variety of lakes, the nearly continuous series of gradations between major types, and the large number of anomalous lakes which don't seem to fit into any category would seem to support this possibility. Larkin and Northcote (1958) found the above phenomena discouraging in their attempts to classify British Columbia lakes:

> The analyses of lake survey data from British Columbia can only discourage attempts to classify lakes as types. Just as there is no sharp distinction in the quantities of total dissolved nutrients, or types of morphometry of classes of climate, ...there is no sharp delineation of lake types. Variation is continuous in both physical determining factors and the biological crops with which they are associated.... the majority share no such harmony in their physical background and paradox is the rule rather than the exception.

However, these workers used only simple regression models (e.g. plankton standing crop vs. total dissolved solids), whereas more sophisticated multiple regression and multi-variate techniques stet the classification problem. Pennak (1958) likewise concluded that a simple trophic classification was not applicable to northern Colorado lakes because of numberous variable, intergrading and anomalous lake conditions. He proposed an altitudinal division into plain, foothill, montane and alpine zones, combined with certain chemical characteristics, as a realistic classification of lake types in this region.

Margalef (1958) and others have noted that classical lake typology is based on a hybrid concept; i.e. it considers the chemical nature of the water, lake basin morphometry, nature of the standing crop, and productivity. It is not always possible to define eutrophy in terms of productivity in a manner consonant with a definition in terms of standing crop (indicator organisms) or of the chemical nature of the water. Zafar (1959) and others have proposed to use the words "oligotrophic" and "eutrophic" to signify only the chemical nature of the lake water, which has disadvantages as well as advantages. Zafar further suggested that lake classification consist of three components: (1) the chemical nature of the water, (2) climatic zone and basin morphometry, and (3) dominant class of organisms and productivity. The suggestion has merit since classical trophic categories do include too many characteristics which are not necessarily correlated. Selection of appropriate quantifiable criteria for a given component of the classification system remains a problem, and if improperly developed, Zafar's system could lead to a proliferation of generated and imaginary lake types.
As a system for routine nomenclature, Zafar's proposal is unwieldy. For example, Anderson-Cue Lake (Putnam County, Florida), a lake under study by the author, would be named as δ - oligotrophic δ - subtropico - absolutae

chrysophyceae. In spite of its deficiencies, this sytem seems to offer an approach to solution of the lake typology problem.

A substantial literature exists, largely in European journals, on the topic of a lake typology. General aspects of lake typology have been reviewed by Elster (1958, 1962), Faegri (1954), Jarnefelt (1952), Macan (1958), Okland (1964) and Thienemann (1955) among others. A considerably larger body of studies are referenced in these papers, indicating much effort on the part of limnologists. However, the subject still has not advanced much beyond semantic arguments into discussion of quantifiable bases for trophic state.

In summary, lakes are extremely heterogeneous in character and defy simple categorization. For example, lakes can be classed in at least 76 categories according to origin alone (Hutchinson, 1957). This is not to suggest that the number of the trophic states and successional patterns should parallel the number of lake basin productive forces, but it does imply the diversity of lakes, particularly on a global scale. As knowledge of lakes becomes more complete, we will undoubtedly be able to delineate many more distinct trophic types and successional patterns than in the present simplistic scheme, and more importantly provide some quantifiable bases for ranking within each trophic classification.

POSSIBILITIES AND LIMITATIONS OF MODELING EUTROPHICATION: GENERAL CONSIDERATIONS

The previous discussion has shown that the problem of eutrophication is complicated. Natural and human factors in lacustrine nutrient enrichment are multitudinous; environmental parameters affecting trophic state are beyond simple description. In addition, eutrophication has ramifications beyond the purely technical. In lake management and eutrophication control, socioeconomic and political aspects necessarily must be considered. In effect, the eutrophication problem is not one but many, and in order to solve it the efforts of many disciplines must be utilized. It is likely that solutions to the problem and its ramifications will require a unified but somewhat simplified analysis of its diverse aspects.

The techniques of systems analysis, or operations research, are especially appropriate for solving complex problems. The concept of systems analysis is hardly recent, but its utility as an analytical and predictive tool has been greatly advanced by development of high speed computers. One of the fundamental principles of systems analysis is that complex problems or systems consisting of many variables can be divided into a number of simpler components each containing relatively few variables. The small components are more amenable to detailed analysis. Solutions to the overall problem can be attained in essence by integrating the partial solutions. A second principle of systems analysis is the concept of the model. A model is simply an approximation of the real system. It consists of the essential system elements and the variables which affect their interactions. Quantitative system models are constructed from various types of deterministic and stochastic mathematical expressions. With the advent of high speed computer techniques, systems analysts have been able to construct sophisticated analytical and simulation models with the many components and variables needed to describe involute real systems.

System analysis techniques and mathematical modeling have been widely applied to complicated industrial and governmental management problems.

Recent developments in water resources management have been closely associated with these techniques, and some applications have also been made in water quality management (primarily regarding models of dissolved oxygen in polluted streams). Ecosystems being characterized by their labyrinthine natures, modeling techniques have been increasingly utilized in ecological studies (e.g. Watt, 1968). The tools of systems analysis and mathematical modeling should also be invaluable to investigators of complex eutrophication problems. A few models have appeared in the literature specifically related to eutrophication; hopefully this workshop will stimulate many more.

The nature of a mathematical model is largely dictated by the scope of the problem and the type of answers one is seeking. As discussed above, eutrophication has numerous technical, social and political facets, and it would be both unrealistic and inefficient to include them all into one super-model. Thus we are talking not about one model but many; various aspects of the problem may require different types of models (e.g. stochastic, analytical, simulation, optimization) depending on the outputs desired. However, any useful model must satisfy at least three criteria:

1) The model must be tractable from a mathematical and computer viewpoint.

2) Except for theoretical studies, a model should require only data that is available, that can be acquired with a reasonable effort, or can be simulated.

3) The model should have proper sensitivity, i.e. the output function should respond to the controlling input variables but should not be affected significantly by extraneous variables. On the other hand, a highly sensitive model may tend to be unstable.

Of the three criteria, probably the second will be the most difficult to satisfy for models of eutrophication.

The critical trophic criteria vary with the degree of eutrophication; for example, taste and odor problems occur at lower nutrient levels than algal scum problems. Thus models concerned with low levels of eutrophy may have different outputs than models of hypereutrophy. Eutrophy associated problems also vary with intended water use. Contact sports required a higher quality water than waters used only for fishing. These facts should be considered in developing models for specific lakes, viz. it makes little sense to develop a model responsive to taste if a lake is not to be used for water supply.

The time scale of the input and output functions must also be considered. Are daily, weekly or monthly averages satisfactory? Controlling factors undoubtedly become more complex as higher frequency oscillations in trophic conditions are considered. One of the difficulties in modeling the effects of nutrient enrichment is the fact that some of the most dramatic effects are characterized by high frequency changes. For example, short term algal blooms are the serious problem, especially in temperate lakes, rather than the average annual standing crop. Dynamic models of trophic state will have to consider these high frequency oscillations in some way if meaningful descriptions of eutrophying effects are to be obtained.

In developing eutrophication models, the possibility that the process of nutrient enrichment and the nature of trophic criteria may vary geographically should be kept in mind. A eutrophication model equally applicable to mountain lakes in Colorado and lowland lakes in Florida may be an unrealistic goal.

Physical processes undoubtedly play important roles in synthesizing a lake's trophic state. Hydrodynamic phenomena, e.g. wind-driven currents and eddy diffusion, affect nutrient and organism transport and mixing. Thermal stratification diminishes nutrient recycling by trapping sinking organic nutrients in the hypolimnion. Organisms like zooplankton may themselves transport nutrients in stratified lakes through diurnal vertical migrations above and below the thermocline. The size of a lake will certainly affect the complexity of model development and the role of physical processes in the model. In small shallow lakes such as the majority of lakes in Florida, neither thermal stratification nor transport processes may be relevant, and lake averages may be adequate as model inputs and outputs. In large and deep lakes, prediction of water quality in localized areas would require knowledge of physical processes (currents and diffusion), especially if small time scale (e.g. daily or weekly) changes are desired as outputs. On the other hand, even for large lakes, time and spatially averaged data may be useful for eutrophication models with longer time scale outputs.

SOME AREAS FOR POSSIBLE DEVELOPMENT OF EUTROPHICATION MODELS

Nutrient Enrichment or Eutrophication Rate Functions

Eutrophication is the process of nutrient enrichment and as such can be defined in terms of the net nutrient flux to a lake. Nutrient enrichment results from both natural and cultural sources; geochemical, hydrological and climatic factors affect the former, and economic, aesthetic and other factors affect the latter. Nutrient budgets require much time and effort, but no intrinsic difficulties are involved in their calculation. As nutrient concentrations in various sources (e.g. rainfall, dometic sewage, urban runoff, etc.) become better known and the factors affecting these concentrations become better understood, it should be possible to model lake nutrient enrichment in terms of certain measured environmental parameters (amount of rainfall, population within the drainage basin, percent land usage in various categories, etc.). Obviously the complexity of a nutrient enrichment model will vary according to the size of a lake and its surrounding basin. For small lakes and basins a simple stochastic model might be sufficient. Large lakes and basins like Lake Erie may require treatment on matrix form, i.e. submodels in a two dimensional grid scheme.

Inevitably, eutrophication involves enrichment of lakes, with essential plant nutrients, whether such enrichment is caused by increased cultural stress and direct nutrient additions (e.g. via sewage effluents), by geochemical and hydrologic changes within the drainage such as occurred in Lago de Monterosi (Hutchinson, 1969), or by internal lake changes, such as increased nutrient exchange with sediments as a lake becomes shallow and unstratified. It may be possible to derive a function such that a given flux of the essential (limiting) nutrients defines a unique rate of eutrophication. As an approximation, eutrophication might be represented by some combination of nitrogen and phosphorus input, assuming trace elements do not limit lake productivity over long periods of time.

While many elements and compounds (e.g. trace metals and vitamins) are essential for algal growth, there is reason (but no unambiguous proof) to believe that lakes generally are limited by the N and P inputs. It is also possible that input of other essential nutrients is correlated with one or both of the major limiting elements. Since organisms require both nitrogen and phosphorus in order to grow, it would seem that the function would not be simple additive, i.e. $F(k_1N + k_2P)$, for this would imply that the requirements of organisms for either element could be replaced by adding more of the other element. A proper function in this case would seem to be multiplicative $(E - F(NP))$ or to at least contain a multiplicative (i.e. second-order) term showing the dependence of the process on both elements. Several simple eutrophication functions could be considered, for example:

$$E = kNP, \qquad (1)$$

$$E = kN^nP^m, \qquad (2)$$

$$E = k_1 N + k_2P + k_3NP. \qquad (3)$$

Obviously, more complicated expressions could be developed. Equation 3 allows positive eutrophication when either N or P input is zero but defines E as zero when both N and P are zero. This is analogous to having a lake high in phosphorus but low in nitrogen, in which case addition of nitrogen alone would cause a certain amount of eutrophication. However, limiting cases where one element has a near zero input and the other a high input are unlikely, and a simple expression like Equation 1 may be satisfactory.

If phosphorus input rate is plotted versus nitrogen input rate for a number of lakes, the product of the two rates defines a certain eutrophication rate according to the simple model given in Equation 1 and lines of equal eutrophication rates can be drawn on the graph. The nature of this eutrophication function implies that the lines of equal eutrophication rates are hyperbolic, and that for a given eutrophication rate, the total nitrogen and phosphorus input rate is a minimum at some N/P ratio, as shown in Figure 4. Lines of varying slope passing through the origin or Figure 4 represent N/P loading ratio. At the optimum N/P ratio, eutrophication proceeds most rapidly for the smallest increase in total N and P input. An arbitrary optimum N/P ratio of 10 is shown in Figure 4; the optimum ratio in the lacustrine environment may be as high as 15 or 20 to 1 (Sawyer, 1966). If the N/P ratio is not optimal, eutrophication could be increased simply by adding more of whichever nutrient is in shorter supply, relative to the optimum ratio.

Undoubtedly the above discussion is unrealistically simplified. It is possible that other nutrients should be considered or that only one of the nutrients need be considered in certain circumstances. For a given total nutrient influx, net enrichment and its effect on trophic conditions may vary depending on the temporal variations in the input (whether it is a steady addition or a periodic slug, and at what season the slug inputs occur). It also seems clear that the eutrophication function must have some time dimensions; certainly a given amount of nutrients would affect a lake differently if fed over a period of years than if it were fed in one year. Perhaps the eutrophication function should be expressed in terms of nutrient quantity per unit time per area of volume of the lake (e.g. lg. N or P per acre per year).

Dynamic Models of Aquatic Ecosystems

The effects of nutrient enrichment on the aquatic ecosystem depend on a variety of edaphic factors. Ecosystems can be described as multi-compartment models (each compartment representing a trophic level or nutrient or energy reservoir), and the interactions among the compartments can be defined by series of differential equations. Environmental control of exchanges between ecosystem compartments can be indicated in such models by appropriate forcing functions or by transfer coefficients which could be functions of certain environmental conditions. Effects of nutrient enrichment on the dynamics and equilibria of aquatic ecosystems could be studied by appropriate in situ experimentation and analysis of the data with suitable dynamic ecosystem models. Much theoretical work could also be done with such models using artificial or computer generated data. Patten (1966), Odum (1960, 1967), and Williams (1969) have derived and discussed models of this type. Brezonik (1968) presented a simple general compartmental model (Figure 5) for the flow of a single resource (e.g. carbon or nitrogen) through an aquatic ecosystem and discussed applications and limitations of such models in eutrophication research. The model in Figure 5 divides the aquatic ecosystem into seven somewhat arbitrary compartments (e.g. phytoplankton, fish, dissolved and suspended organic matter). The model could be made more complicated by further subdividing some of the compartments. For example, fish could be divided into herbivores, carnivores, and omnivores, or according to physical habitat into littoral and pelagic compartments. The arrows connecting compartments represent pathways of resource flow (e.g. zooplankton (B_4) grazing of phytoplankton (B_2) is indicated by the arrow $T_{24}B_2$, and by which the resource (nitrogen, carbon, etc.) is transfered from B_2 to B_4. The final interrelated scheme of compartments inevitably reflects the developer's view of the real world or prototype. A model is not an isomorph of the prototype, i.e. there is not a one to one correspondence between model and prototype elements. There are fewer distinct elements in the model than in the prototype, and there is a certain subjectivity involved in combining distinct elements of the prototype into a single model element and in deleting other prototype elements altogether from the model.

The elegance and realism of compartmental ecosystem models are largely dependent on the functional relationships between compartments and on the constraints placed on transfer coefficients (e.g. whether they are constant or time dependent). Probably the simplest model (and also the most unrealistic) is one where compartmental inputs are all precursor first order, i.e. the rate of transfer from one compartment to another is proportional to the size of the precursor (donor) compartment. Table 4 presents simultaneous differential equations which define the model in Figure 5 according to first order precursor-dependent relationships. The rate of change in the size of a given compartment equals the sum of the ratio of inputs from donor (precursor) compartments minus the sum of the rates of transfer from the compartment to other trophic levels. For example, in words the rate of phytoplankton compartment change (B_2) (in terms of nitrogen, carbon, etc.) equals the rate of assimilation from inorganic sources (which is photosynthetic dependent) minus the rate of herbivore grazing and the rate of mortality. The equations in Table 4 can be solved by computer with a simple finite difference program to yield compartment sizes at any given time, provided that values can be placed on the various transfer coefficients and forcing functions.

While some transfer rates in the above model may realistically be represented as first order precursor-dependent, other rates (e.g. those involving predators like fish) would seem to depend on the size of the receiving compartment. Some transfer relations are probably second order in nature, i.e. dependent on the sizes of both precursor and receiver compartments. Second order transfer functions may be further complicated by a self-inhibition factor of the acceptor compartment. The Lotka-Volterra function is such a second order transfer relation with acceptor self inhibition, i.e. as the acceptor compartment increases in size it exerts an

TABLE 4

First Order Differential Equations Describing Single Resource Ecosystem Model in Figure 5.

$$\dot{B}_1 = Q_1 + T_{41}B_4 + T_{61}B_6 + T_{71}B_7 - \Pi_{12}B_1 - \Pi_{13}B_1;$$

$$\dot{B}_2 = \Pi_{12}B_1 - T_{24}B_2 - T_{25}B_2 - \mu_{26}B_2 ;$$

$$\dot{B}_3 = \Pi_{13}B_1 - T_{34}B_3 - \mu_{36}B_3 ;$$

$$\dot{B}_4 = T_{24}B_2 + T_{34}B_3 + T_{64}B_4 - T_{41}B_4 - T_{45}B_4 - \mu_{46}B_4;$$

$$\dot{B}_5 = T_{45}B_4 + T_{25}B_2 + T_{65}B_6 - \mu_{56}B_5 ;$$

$$\dot{B}_6 = \mu_{26}B_2 + \mu_{36}B_3 + \mu_{46}B_4 + \mu_{56}B_5 - T_{61}B_6 - T_{64}B_6 - T_{65}B_6 - T_{67}B_6;$$

$$\dot{B}_7 = T_{67}B_6 - T_{71}B_7$$

B_i = biomass or total nutrient content in trophic level i;

Π_{ij} = fractional rate of primary production (as organic carbon, biomass, or organic nutrient synthesized);

T_{ij} = fractional rate of biomass or nutrient transfer between trophic levels or non viable units;

μ_{ij} = mortality coefficient, or fractional transfer rate of biomass or nutrient from viable to nonviable compartment.

Q_1 = total nutrient or organic matter input rate to ecosystem;

inhibiting effect on the rate of transfer. This type of function reflects over-crowding effects found in predator-prey relationships in nature and is usually necessary in models in order to impart stability. Transfer functions from inorganic to living compartments more likely are given by Monod or Michaelis-Menten type expression than by simple first order kinetics. These possible functional relationships between compartments are summarized in Table 5.

It is also likely that transfer coefficients are not constants but vary according to the environmental factors, e.g. temperature, light and nutrient levels. Seasonal variations in temperature and light can be approximated by sinusoidal variations and normal distributions, but seasonal nutrient variations are usually more complex. However, (inorganic) nutrient effects would seem to be limited mostly to the primary production reactions, and their significance could be assessed by multiple regression methods (see 4 below). Within limits, light intensity has a linear effect on primary production rates; temperature has a logarithmic effect within the range of the organisms thermal tolerance.

Alternatively one could attempt to develop realistic dynamic (differential equation) models for certain components of the ecosystem without trying to solve simultaneous equations for all ecosystem compartments. For example, phytoplankton biomass can be written as a function of primary production, grazing, mortality, sinking, etc. Functional relationships are known or can be estimated for most of the major environmental factors affecting phytoplankton population dynamics. The differential equation can be written in terms of readily measurable physical and chemical factors (light, temperature, nutrients, turbulence) except for herbivore grazing

TABLE 5

Possible Functional Relationships Between Compartments in Ecosystem Models

$$\boxed{B_i} \xrightarrow{T_{ij}} \boxed{B_j}$$

First order (linear) precursor: $dB_j/dt = T_{ij}B_i$

First order (linear) acceptor: $dB_j/dt = T_{ij}B_j$

Second order (non-linear; first order in each compartment): $dB_j/dt = T_{ij}B_iB_j$

Michaelis Menten or Monod: $dB_j/dt = \dfrac{T_{ij}B_jB_i}{K_{Tij}+B_i} = \dfrac{V_{ij}B_i}{K_{Tij}+B_i}$

where $V_{ij} = T_{ij}B_j$ = maximum rate of transfer (at B_i saturation)

Lotka - Volterra relation $dB_j/dt = T_{ij}B_iB_j(1 - \alpha_jB_j)$

where α_j = coefficient of self inhibition by acceptor compartment which involves knowledge of zooplankton biomass, growth rates, and yield coefficients.

It should be obvious from the preceding discussion that realistic ecosystem compartment models will be rather complicated, involving many types of functional relations and necessitating large computers for solution. More importantly, the data requirements for such models are also rather excessive and probably will limit their usefulness.

A Trophic State Equation

One major limitation of compartment models for eutrophication research relates to the nature of their outputs, i.e. compartment sizes (biomass in various trophic levels) as a function of time. While these are of unquestioned interest and importance to ecologists, except for phytoplankton biomass, they do not directly relate to the critical parameters in eutrophication phenomena, i.e. water quality criteria. Models which relate nutrient and environmental inputs to water quality outputs like bloom frequency and intensity, taste and odor problems, water transparency, and the like may be more pertinent to the immediate problem of eutrophication.

As previously noted, trophic state is defined imprecisely and mostly in qualitative terms. No single parameter defines trophic state, but it is a summation of many biological and chemical conditions. It may be useful to develop quantitative definitions of trophic state even if the definitions are somewhat arbitrary. One possible avenue would be to develop a trophic state parameter as some simple function of the trophic indicators shown in Table 3. The nature of this function is speculative but could involve the following variables, for example:

$$\text{Trophic State} = F\text{ (Primary production, algal biomass, Secchi disc transparency, nutrient concentrations).} \qquad (4)$$

The relative importance of the different variables, the time scale, units of the variables, and their functional relationships would require considerable thought and development. Also, a universal trophic state function for all lakes in all geographical regions may not be a realistic goal. Simple regression analyses could not be used to develop such functions since only the independent variables are known while the dependent variable (trophic state) is what we are seeking to define. A possible approach would be to define a series of trophic state equations arbitrarily and determine by canonical analysis which equation gives the highest correlation with a defined nutrient input or eutrophication function. Other multivariate methods may also be suitable tools to develop trophic state functions; a later panel will discuss these in greater detail. Such trophic state functions could be used for more realistic trophic classification schemes. If properly developed, they could also be used to indicate probabilities of specific water quality problems, such as taste and odor, swimmer itch, filtration problems, from the standard trophic indices.

Factors Affecting Primary Production

The importance of primary production in eutrophication and determination of lake trophic state can hardly be overemphasized. Thus studies on the environmental factors controlling primary production have important implications in the present problem. These factors include physical variables like temperature and light, chemical variables such as nutrient concentrations, pH, ionic balance, and biological factors such as predation, competition and symbiosis. A variety of bioassay techniques are available to determine the nutrient or nutrients limiting production. These techniques are useful in eutrophication to determine the elements least available to the algae (relative to their need), which therefore are most critical in the eutrophication process. However bioassay methods are not directly applicable to physical factors like light and temperature. One way of evaluating the array of limiting environmental factors is by statistical (e.g. multiple regression) and correlation analysis of extensive field data on primary production and simultaneously measured environmental conditions. This method has been applied to productivity studies in Lago Maggiore, Italy, By Goldman _et_. _al_. (1968) and in some small Florida lakes by Brezonik _et_. _al_. (1969). In the former study, light was found to be the most important factor controlling primary production. A step-wise multiple regression analysis of primary production against 12 environmental variables gave a multiple correlation coefficient of 0.909. The correlation coefficient can be interpreted as the square root of the portion of variance in the independent variable accounted for by the independent variables. Thus the 12 variables accounted for $(.909)^2 = 82.6\%$ of the variance in primary productivity. However, only the first five independent variables (light, temperature, oxygen, species diversity, and silicate) contributed significantly to the multiple correlation coefficient. In an intensive short term (10 day) study of primary production in Anderson-Cue Lake, Florida, Brezonik _et_. _al_. (1969) found that primary production regressed against light intensity (as indicated by underwater photometer readings) yielded a simple correlation coefficient of 0.76, while a multiple regression of primary production against light intensity, water temperature, dissolved oxygen, pH, acidity, ammonia, nitrate, ortho phosphate, and chlorophyll *a* yielded a multiple correlation coefficient of 0.87 (Table 6). Thus light alone explained 58% of the variance in primary production while the nine variables (including light) accounted for a total of 76% of the variance. Multiple regression analysis of primary productivity data from routine studies on this lake over a period of 21 months has shown a strong correlation with nutrient concentrations. For example, a regression of surface primary production vs. total phosphate and total organic nitrogen gave a multiple correlation coefficient of 0.73, while a regression of surface primary production vs. eight nutrient and one physical variable gave a multiple correlation coefficient of 0.93 (Table 6).

The method of multiple regression analyses suffers from many statistical limitations and is inherently empirical. It is premature to say whether equations derived by stochastic techniques to predict primary productivity from environmental factors will have general validity or will apply only to local conditions. Nevertheless, the method is attractive, and in the absence of any analytical or deterministic models for lacustrine primary production, stochastic models are the only realistic alternative.

Economic Analysis of the Eutrophication Problem

There are many degrees of eutrophication and its manifestations, many alternatives for their control and prevention, and only finite resources available to do the job. Thus, it becomes a highly involved problem to determine how much money and other resources to spend for its prevention, control, or alleviation. This is an optimization problem in which we

TABLE 6

Multiple Regression Analysis of Primary Production Controlling Factors in Anderson-Cue Lake, Florida: Some Examples[1]

Study[2]	Independent Variables[3]	N[4]	Regression Equation	Multiple Correlation Coefficient[5]
10 day May, 1968	Pho	90	$PP = 1.82 + 1.23(PO)$	0.76
	Pho, CHA, CO, AC, pH, WT, PO_4, NH_3, NO_3^-	90	$PP = -10.1 + .98(PO) + .31(CHA) + .70(DO) + .75(AC) - 20.74(NO_3^-) + .13(WT) - .27(NH_3) + .07(pH) + 2.00(PO_4)$	0.87
Routine Monthly Sampling	TP, TON	21	$PP = -6.00 + 4.18(TP) + 25.20(TON)$	0.73
	TP, PO_4, TON, NH_3, WT, DO, pH, AC, SOLR	21	$PP = -436.51 + .38(TP) + 140.29(NH_3) + 1.26(PO_4) + 72.76(TON) + 13.33(pH) + 5.09(WT) + 24.03(DO) + 10.00(AC) + .02(SOLR) + 63.16(NO_3)$	0.93
	NH_3, NO_3, PO_4, SOLR	21	$PP = 3.1 + 121.7(NH_3) + 2.90(PO_4) + 87.0(NO_3)$	0.82

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[1] From Brezonik, Morgan, Shannon and Putnam (1969).

[2] 10 day study includes samples from 3 depths: surface, 5 and 10 feet, routine data are for surface requirements only.

[3] Pho = underwater photometer reading; CHA = chlorophyll a; DO = dissolved oxygen; AC = acidity; pH = $-\log(H^+)$; WT = water temp.; TP = total phosphate; PO_4 = ortho phosphate; TON = total organic nitrogen; NH_3 = ammonia; NO_3 = nitrate; PP = primary production; SOLR = total solar radiation.

[4] N denotes sample size.

[5] All regressions significant at the 99% confidence level.

desire to maximize the benefits at a given cost or minimize the cost at a given level of benefits. Optimization is a familiar problem in systems analysis. Involved are cost-benefit analyses, i.e. for a given amount of resources spent to remedy the problem, what and how many benefits accrue. One complicating factor with respect to eutrophication management is evaluation of recreational and intangible aesthetic beneficial uses associated with oligotrophy and mesotrophy. Further work on this aspect may have to await clearer definitions of the eutrophication process and its effects, and answers to some of the more basic questions discussed above.

CONCLUSIONS

Undoubtedly many other possibilities exist for application of mathematical models in eutrophication research and control. The above comments represent the thoughts of the author as tempered by discussions with many colleagues over the past few years. Hopefully these thoughts will stimulate others to develop suggested models more fully and to derive new models and fresh approaches.

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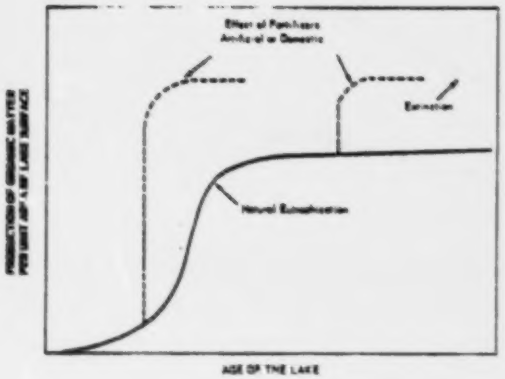

Figure 1. Hypothetical Curve of the Eutrophication Process in Lakes. (After Hasler, 1947).

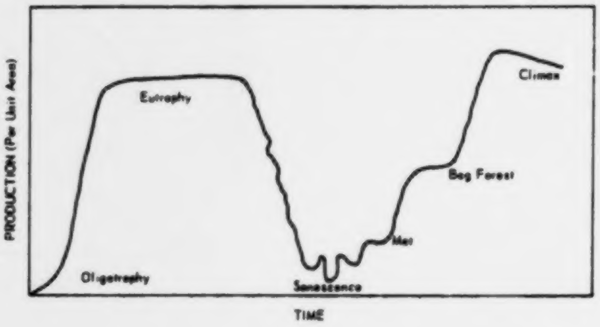

Figure 2. Lindeman Model of Trophic succession and Extinction of Lakes. (After Lindeman, 1942).

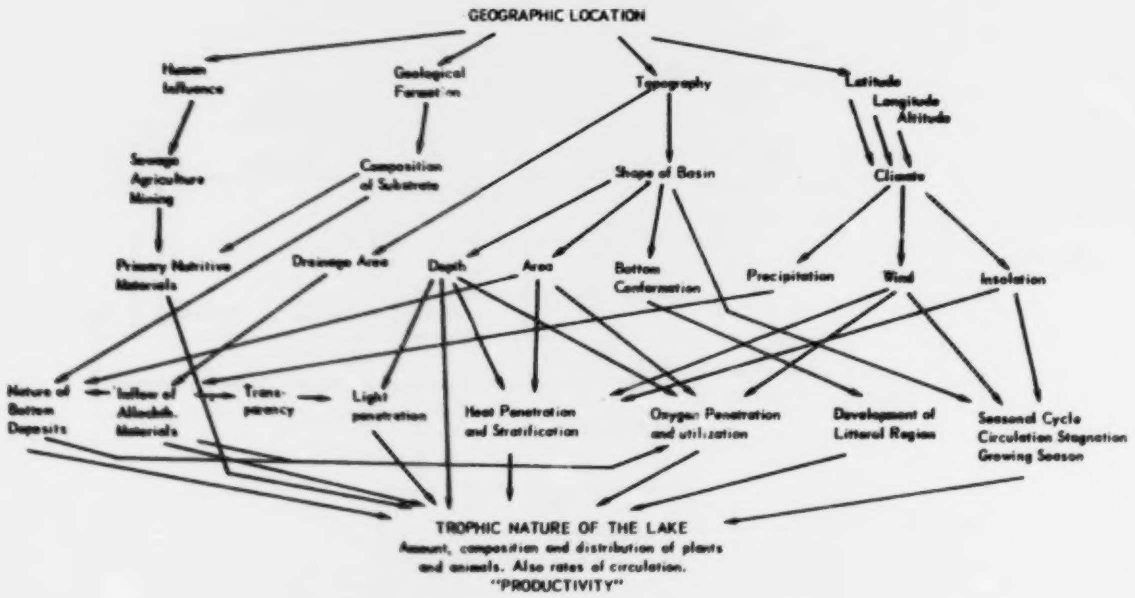
GEOGRAPHIC LOCATION
Human Influence
Geological Formation
Topography
Latitude
Longitude
Altitude
Sewage Agriculture Mining
Composition of Substrate
Shape of Basin
Climate
Primary Nutritive Materials
Drainage Area
Depth
Area
Bottom Conformation
Precipitation
Wind
Insolation
Nature of Bottom Deposits
Inflow of Allochth. Materials
Transparency
Light penetration
Heat Penetration and Stratification
Oxygen Penetration and utilization
Development of Littoral Region
Seasonal Cycle Circulation Stagnation Growing Season
TROPHIC NATURE OF THE LAKE
Amount, composition and distribution of plants and animals. Also rates of circulation.
"PRODUCTIVITY"

Figure 3 . Chart suggesting the interrelationships of factors affecting the metabolism of a lake. (After Rawson, 1939).

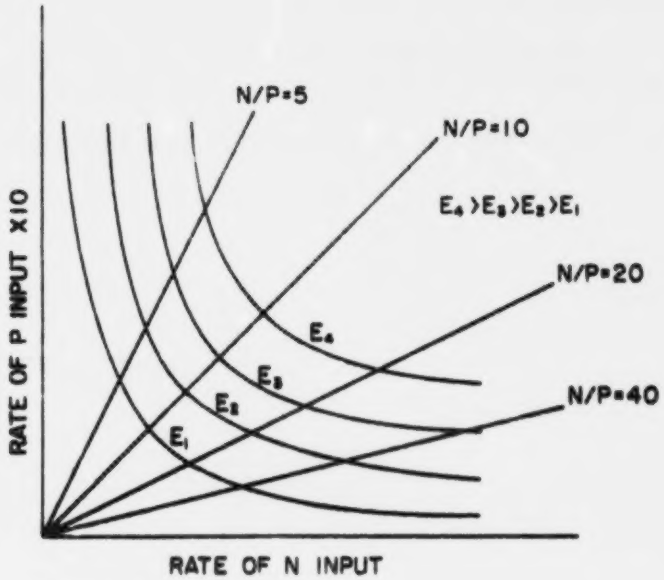

Figure 4. Hypothetical rate of eutrophication as a function of nitrogen and phosphorus loading rates. Rate of eutrophication given by $E=k(NP)$ In the case shown, optimum loading for eutrophication occurs at N/P=10

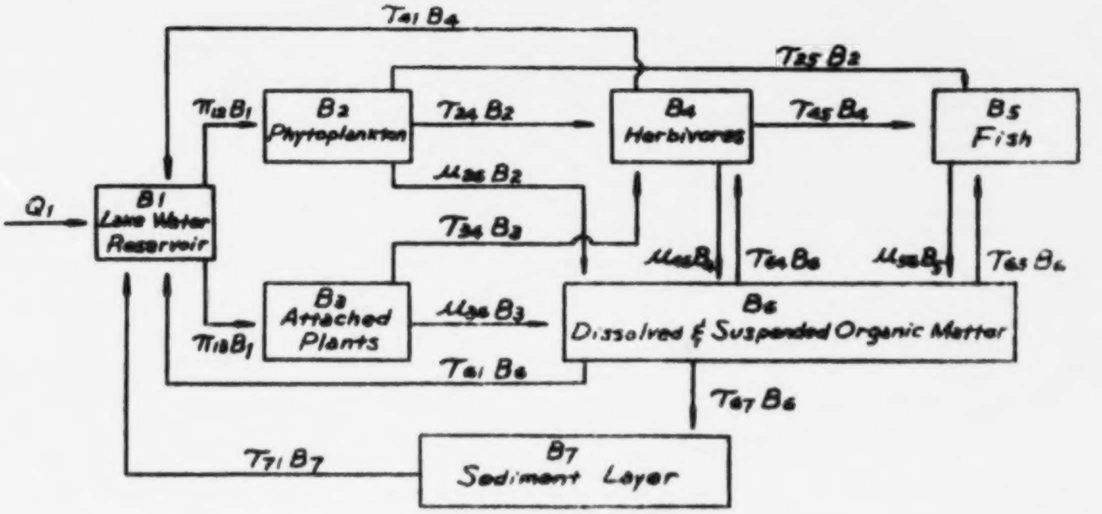
$\tau_{41} B_4$
$\tau_{25} B_2$
$\pi_{12} B_1$
B_2 Phytoplankton
$\tau_{24} B_2$
B_4 Herbivores
$\tau_{45} B_4$
B_5 Fish
$\mu_{26} B_2$
Q_1
$B1$ Lake Water Reservoir
$\tau_{34} B_3$
$\mu_{46} B_4$
$\tau_{64} B_6$
$\mu_{56} B_5$
$\tau_{65} B_6$
B_3 Attached Plants
$\mu_{36} B_3$
B_6 Dissolved & Suspended Organic Matter
$\pi_{13} B_1$
$\tau_{61} B_6$
$\tau_{67} B_6$
$\tau_{71} B_7$
B_7 Sediment Layer

Figure 5. Single Resource Ecosystem Model (from Brezonik, 1968).

EUTROPHICATION: THE PROCESS AND ITS MODELING POTENTIAL

A Summary Statement

by

The Panel Members

EUTROPHICATION: THE PROCESS AND ITS MODELING POTENTIAL

There was general agreement among workshop participants concerning the need for a more precise definition of eutrophication, but various opinions were expressed concerning the nature of this definition and its purpose and benefits. Some felt that eutrophication should be described by its symptoms, whereas others felt the term refers to the process. Several argued that the definition of eutrophication must be related to its effects on man. The consensus of the panel is that eutrophication and terms associated with it are broad qualitative terms and probably should remain so. There should be a simple definition of eutrophication, agreeable to scientists and understandable by the layman. The definition related by Dr. Bartsch from the International Organization for Economic Cooperation and Development seems to fulfill this need. According to this definition, eutrophication is the nutrient enrichment of waters resulting in an array of symptomatic changes, among which are production of algae and other aquatic plants, the deterioration of fisheries, the deterioration of water quality, and other changes, which are found objectionable and interfere with water use.

If eutrophication cannot be precisely defined in quantifiable terms, it follows that eutrophication per se cannot be modeled. Whether this is important or not is another matter; we can model various aspects of the problem insofar as we can quantitatively define them and relate them to nutrient enrichment. The central phenomenon in eutrophication is the high rate of production, which in general results from some sort of nutrient enrichment. Various consequences of high production rates can be distinguished according to morphometric and other physical circumstances.

Cultural and natural eutrophication are viewed as distinct processes with different causes and perhaps somewhat different effects. There seems little question that natural eutrophication is a real process, but whether it always or even typically occurs according to the classical Hasler-Lindeman model is another question. Some lakes may become eutrophic shortly after their formation as a result of readily leachable nutrients in their drainage basin and may then proceed toward oligotrophy. Other lakes may be eutrophic throughout their life-histories. Lakes which become eutrophic according to the Hasler-Lindeman model do so at least partially as a result of their gradually becoming more shallow, as Hasler pointed out in his much cited 1947 paper. On the other hand, cultural eutrophication can occur without significant changes in a lake's depth. It usually results from direct addition of nutrients by human activity into the lake or into the lake's sources of water. However, artificial changes in hydrology and basin geology can indirectly cause nutrient enrichment of lakes as well. Cultural eutrophication is probably a more severe problem than natural eutrophication; the former is apparently reversible and controllable at least to some extent and will probably receive greater attention in modeling efforts.

The panel feels that modeling has much potential for solution of eutrophication problems. One of the major benefits of modeling is the organization it provides to our thinking and the fresh insights it provides into familiar problems. Modeling should be a useful tool in solving specific complex management problems related to eutrophication. At the present time it seems possible to develop analytical models of lacustrine production as functions of nutrient levels and environmental factors.

Eventually it would be desirable to model specific problems such as taste and odor, algal scum, filter clogging, etc. Eutrophication is of concern because of its detrimental effects on intended water uses. Primary production itself is not the problem, but to some extent it is the cause of quality problems related to water use.

The problems generated by nutrient over-enrichment of natural waters can be considered in three main categories: water treatment, recreation, and agriculture; Table 1 summarizes the major problems in each of these areas. In general eutrophication causes water quality degradation such that water treatment costs are increased, and agricultural and recreational uses are decreased, impaired or made more expensive. The relationship between nutrient enrichment and the problems in Table 1, or more directly between lake production and these problems can be approached by stochastic models in a management context.

There is an urgent need in lake management for answers to some basic questions about eutrophication. Better information on critical variables and critical levels of nutrients are needed by decision makers now. Stochastic or empirical models using multi-variate techniques would seem to be the quickest and most appropriate way to get approximate answers needed to make management decisions.

Three main types of models were discussed during the workshop, and each has an important role to play in solving eutrophication problems. Physical models (ponds, chemostats) are obviously much more easily manipulated than entire lakes, and these models can provide useful data to predict the effects of changes on the prototype. They have an advantage in providing real numbers rather than computer generated data and will undoubtedly find application both in fundamental studies of the problem and in applied studies for predictive purposes.

Emperical models (Purely stochastic regression or multi-variate models) can be used to provide predictions for management where functional (cause-effect) relationships are not known. Empirical models can be used to give clues to the important or controlling variables, even though they cannot prove cause-effect relationships by themselves. For example, multiple regression analysis can suggest the importance of an environmental factor in controlling primary production; controlled experiments could then be devised to evaluate the existence of a causative relation. Multi-variate techniques can also be applied to developing trophic state indices and lake classification schemes.

The third type of model is the mechanistic model, which is also referred to an deterministic or analytical. This type of model seeks to define functional relationships between parameters, i.e. the basic mechanism involved in a particular process. For example phytoplankton population dynamincs can be expressed interms of the basic relationships between photosynthesis and light intensity, temperature nutrient levels, etc. A more sophisticated mathematical model would include a stochastic component in the mechanistic model in order to account for extrema. For example, a primary production model with a stochastic component could be used to predict the probability of wind-induced concentration of an algal bloom on the side of a lake in which a water supply intake is located. This type of model has probably the greatest potential for usefulness in solving complex eutrophication problems.

Table 1

Water Use Problems Resulting From Eutrophication

A. Water Treatment Problems

- Increased color and turbidity in raw water.
- Increased taste and odor may necessitate use of activated carbon.
- Increased chlorine demand.
- Coagulation difficulties.
- Shortened filter runs.

B. Recreational Problems

- Loss of desirable fish and increase in rough fish.
- Increased costs in boat and dock maintenance resulting from scum and fouling.
- Economic losses to resort and fish camps owners as fewer people swim, fish and boat in lakes with algal blooms and scum.
- Public health problems - swimmers diseases.
- General loss in lake's aesthetic appeal.

C. Agricultural Problems

- Transmissibility of water impaired by extensive macrophyte growths in canals.
- Toxicity of algal blooms to cattle and other livestock.
- Serious increases in water loss in arid regions caused by evapotranspiration of dense vegetation.

DISCUSSION

EUTROPHICATION: THE PROCESS AND ITS MODELING POTENTIAL

BARTSCH: There also needs to be a simple definition that many people can agree with, so this might be an appropriate time to point out that a group of eutrophication experts of the Organization for Economic Cooperation and Development, which is headquartered in Paris, and to which Duttweiler and I go on accasion proposed what we called a perfect definition. This was to get Canada and the United States and some ten European countries to agree on what we view eutrophication to be from the more or less semi-layman scientist's viewpoint. If you will permit me, I will search my memory and see if I can repeat it for you. It goes like this: Eutrophication is the nutrient enrichment of waters which results in an array of symptomatic changes among which production of algae and other aquatic plants, the deterioration of fisheries, the deterioration of water quality, and other changes, are found objectionable and interfere with water use.

BREZONIK: There are changes - lakes do die, but they don't die as the result of eutrophication.

LEE: The final step is the macrophyte problem. Is this what produces senescence?

BREZONIK: If you define eutrophication as a process of nutrient enrichment, I think you can raise the question legitimately, because there is no reason that a lake should all of a sudden get an increased influx of nutrients.

SHAPIRO: This is part of the qualitative definition. I think the definition you cited was an increase in nutrients leading to an increase in the abundance of algae, and so on, so an increase in nutrient, per se, is not enough.

LEE: Natural eutrophication is a timed problem. We have problems of algae in lakes which are not related to man. Are these problems caused or dependent on time? Does a lake age? I don't think it stops. I don't think a lake, once the watershed stabilizes, ages at all. Stepwise, now, it ages when it fills, and then you change to a different regime.

BREZONIK: My point of view is that you can break down the problem and deal with two aspects, nutrient enrichment aspect, and the effect of nutrient enrichment on the lake. Obviously they are both related, but if you just look at nutrient enrichment for a moment and consider that as a separate aspect of the problem, then its effect on trophic state is determined by morphometric factors.

SHAPIRO: I think the word eutrophication should very definitely be limited. How are diseases described? You have the cause for the disease and you have the symptoms of the disease. A disease is described by its symptoms, not by the bacterial agents, or what have you, that caused it. I think the word is etiology. I think we are causing an immense amount of confusion, not only in our own minds, but in other people's minds, by using eutrophication in such a broad way. Eutrophication should be a word that is reserved for description of the symptoms or to the consequences of the addition of nutrients, or aging of a lake. Not to the process.

FRUH: One definition for natural eutrophication probably is just the nutrient input that is causing this. Cultural eutrophication is just one of the stresses.

LEE: One thing to think about in modeling eutrophication is that it is not production or average production we are talking about in proposing criteria. It is the severity and frequency of the noxious algae blooms, and our modeling in eutrophication must be related to this question. The fact that this bloom accumulates along the shore and causes the stink is what we are trying to correct. It is not the fact that the water is green.

SHAPIRO: I would argue with that, too, very strongly. I don't think eutrophication should have such a parochial definition. I think the word eutrophic is a productivity synonym.

BEETON: Can I get into this argument now? I certainly agree that algae blooms are the main things we should be concerned with. I am going to be parochial, too, in a way. I am going to pump this problem back to the Great Lakes. The main thing was the change in the fish population, and then the change in water quality. It wasn't so much the algal problem. The thing that popped up first was the change in the fish and then when we looked at it more closely we noted the change in the nutrients.

LEE: The definition of eutrophication must be related to its effect on man.

MORGAN: I would suggest that in trying to get around the hangup, it must be agreed that the central phenomenon in eutrophication is the high rate of production. One can distinguish various consequences of high rates of production involving morphometric situations. Scientifically, a part of every model will be this high rate of production and how we control that.

BREZONIK: A simplified model of eutrophication would consider the production on a fairly long term basis, I would say, a weekly, monthly, or annual basis. But that is not going to solve our problems.

BEETON: I think that you are just going to compound our problem by defining eutrophication on the basis of symptoms. We are going to have a whole category of lakes where you have an increase in nutrients, but perhaps you are not making a measurement, or you are not getting a biological response to the increase in nutrients.

ECHELBERGER: I tend to agree with a number of you people, in talking about this definition of Eutrophication. I think we are missing one very important aspect of it, and I don't know if we can qualify this or not. We can take the consequences that Jim talks about, and then the enrichment that Dr. Bartsch is talking about, but what we've really got to do is try to define how these things are relevant to the uses that these bodies of water can be put to. If we can quantify these various uses, and then quantify these uses to extinction, then we can get a contemporary definition of eutrophication.

VALLENTYNE: What we are looking at are several confusions in this issue which have historical reasons. I think we look back in early literature where people developed these concepts of eutrophication, you can guess at the confusion between eutrophication in the sense of nutrient supply, and on the other hand transference of that into biological production. I think another confusion that we are looking at now is that limnology really is science through us, and for the most part develops logical biological interest, and delineates for us the production or utilization of fish. What people were looking at before science produced these concepts, was looking at a lake for measurements made for the lake's interest, and also the fish that were in the lake. What we are looking at now is in terms of management's interest, monopoly's interest. We are looking at it not so much, perhaps, for the fish, but the over all aspects of water utilization. I thought Jim and Fred's remarks were really pertinent and drove home the problem that really what we are after here is some measure of how these changes are affecting man's use. In that sense, the suggestion that perhaps production may be the best measure we have of predicting. Not production, per se, but these little facts on man's use.

LEE: Now that we are manipulating environment, we are playing games in the Great Lakes in terms of removing phosphorus, I hope we have sufficient money available to evaluate what we are learning.

CONCEPTS AND UTILITIES
OF AN
ECOLOGIC MODEL

A Position Paper

by

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INTRODUCTION

From a cursory analysis of the environmental problems facing cities in general and water resources systems in particular, one finds several things in parallel. In the cities, there are "slums" and occasionally a "summer riot" occurs. Cities may also die and become "ghost towns". Similarly, the pollution of natural and artificial water bodies has given rise to "eutrophication" and riot-like "algal blooms". Eutrophication, which is characterized by the excess growth of algae, detritus accumulation, exhaustion of dissolved oxygen, fishkills and other associated nuisances, has threatened to shorten the life span of the nation's water bodies and to destroy the utility of the water resources.

In the past, the study of eutrophication has been concerned primarily with a fundamental understanding of its component processes, with emphasis on field and laboratory work. "Neat little problems", concerned with a special aspect of eutrophication, have been isolated and studied in a piece-meal approach to the investigation of the total environment.

Recently, the public's concern about responsible stewardship of the nation's water resources has created an intensifying demand on the scientific community for a more enlightened understanding of environmental problems. This demand opens up a new horizon of research with a scope beyond that of any traditional field of science. Now, today and tomorrow, laboratory results gathered under well controlled conditions must be transferred outdoors to predict changes in the "real" ecologic system where parameters vary in time and space. Therefore, a multi-disciplinary background is required to bring the scattered bits and pieces of information that have been accumulated to bear on the broader problem of management of aquatic environments.

This paper describes a method that seeks to integrate the fundamental principles from all fields including chemistry, biology, ecology, limnology, meterology, hydrology, hydrodynamics, and sanitary engineering. Such a method, characterized by a mathematical model of ecologic processes, may serve as a quantitative means of assessing the state of eutrophication, predicting its trends, and evaluating methods for its control.

THE ECOSYSTEM AND ECOLOGIC PROCESSES

Model development begins with a definition of the system. Within the aquatic environment, four major components can be identified:

1. Abiotic Substances which include basic elements and compounds such as H_2O, CO_2, NO_3, PO_4, Fe, and other water quality constituents.
2. Producers which include autotrophic organisms such as algae.
3. Consumers which include hetertrophic organisms such as zooplankton and animals (benthos and fish).
4. Decomposers which include bacteria and fungi.

These components are interrelated through complex ecologic processes. Figure 1 is a schematic diagram showing the basic components of an ecosystem (boxes) and their interrelationships (lines).

The system receives abiotic substances from various external sources including air, soil and adjoining waters. The producers, with the aid of solar energy, manufacture complex organic materials from the abiotic

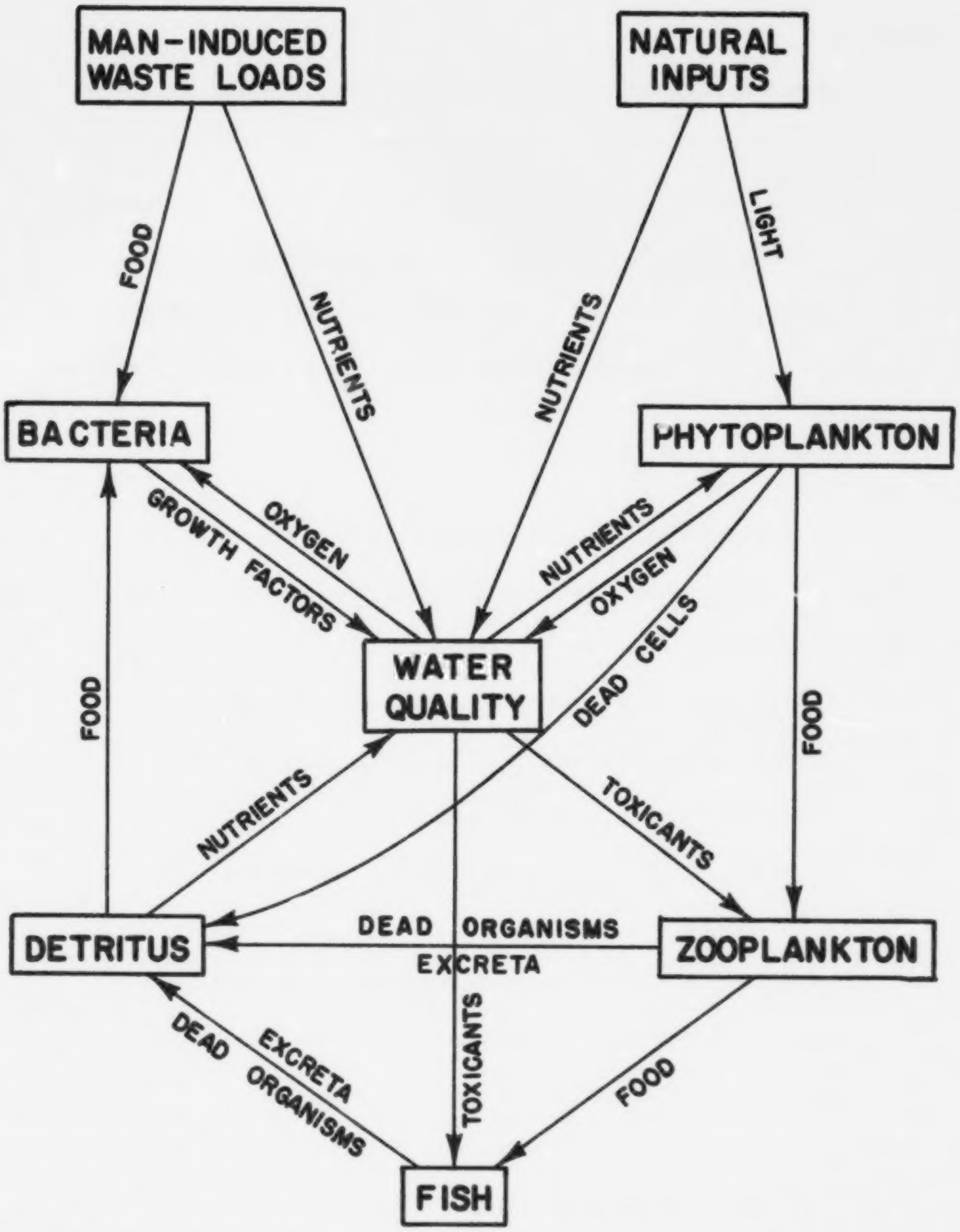
MAN-INDUCED WASTE LOADS
NATURAL INPUTS
FOOD
NUTRIENTS
NUTRIENTS
LIGHT
BACTERIA
PHYTOPLANKTON
OXYGEN
GROWTH FACTORS
NUTRIENTS
OXYGEN
DEAD CELLS
WATER QUALITY
FOOD
FOOD
NUTRIENTS
TOXICANTS
DETRITUS
DEAD ORGANISMS
EXCRETA
ZOOPLANKTON
DEAD ORGANISMS
EXCRETA
TOXICANTS
FOOD
FISH

FIGURE I. DEFINITION OF AN ECOSYSTEM

substances through a photosynthetic reaction. These organic materials serve as primary food sources for herbivorous animals, such as zooplankton, which in turn are consumed by carnivores, such as fish. All of these biological activities generate detritus which consists of dead cell material and excreta or undigested food stuffs. Bacteria and fungi, the decomposers, operate on the detritus to release simple abiotic substances for use by the producers, thus completing the "cycle".

Thus, it is that a water resources system may serve as a giant reactor for the biological cycle. Bacteria oxidize or demineralize organic matter to liberate nutrients, and phytoplankton reduce the minerals and resynthesize the organic materials to support other life. When these two processes are in balance, the rate of eutrophication is at a virtual minimum (Oswald, et. al., 1966). An unbalanced situation tends to stimulate excess algal growth and create nuisances. Likewise, on the bacterial side, an imbalance may tend toward deoxygenation and anaerobic conditions.

Such an imbalance in the ecologic system may result from either one or a combination of two causes: an extraordinarily large organic input to the system, especially in streams or estuaries, and/or thermal stratification in the case of lakes and reservoirs (Fruh, et. al., 1966). Thermal stratification in a reservoir causes the two biological processes to react in separate zones--planktonic activity in the epilimnion and bacterial activity in the hypolimnion. After the seasonal overturn, the nutrient rich water from the hypolimnion is mixed with the epilimnal water, resulting in an enrichment that tips the biological balance.

AN ECOLOGIC MODEL

The ecologic phenomena in the real world have been shown to vary dynamically in space, influence greatly by the hydrodynamic behavior of the aquatic system. A comprehensive ecologic model must therefore describe not only the biological processes, but also the transport phenomena of the water and its quality constituents.

A model with such capability has been envisioned after a literature review of some 100 papers and reports (FWPCA, 1968). The general method of approach will be described briefly in the following paragraphs.

Prototype Geometry

The model represents the water system by a network of discretized "elements" chosen to be representative of characteristic components of the water body. Figure 2 shows such an element as it may be defined for a stream or estuary. For reservoirs or lakes where the primary hydrodynamic concern is stratification, elements can be taken horizontally as shown in Figure 3.

For any given time interval, the element behaves like a completely mixed reactor, allowing the mass of water quality constituents (including biota) to be transported in and out by both advection and diffusion. The "state" of the system is described at any desired time over the space continuum formed by the aggregation of discrete elements.

Hydrodynamic Characterization

The hydrodynamic description of the aquatic environment is a complex problem, for which the ideal solution technique has yet to be developed. However, methods are available for two classes of problems which may be applicable for a great number of water systems.

A hydrodynamic program has been developed to solve the equations of

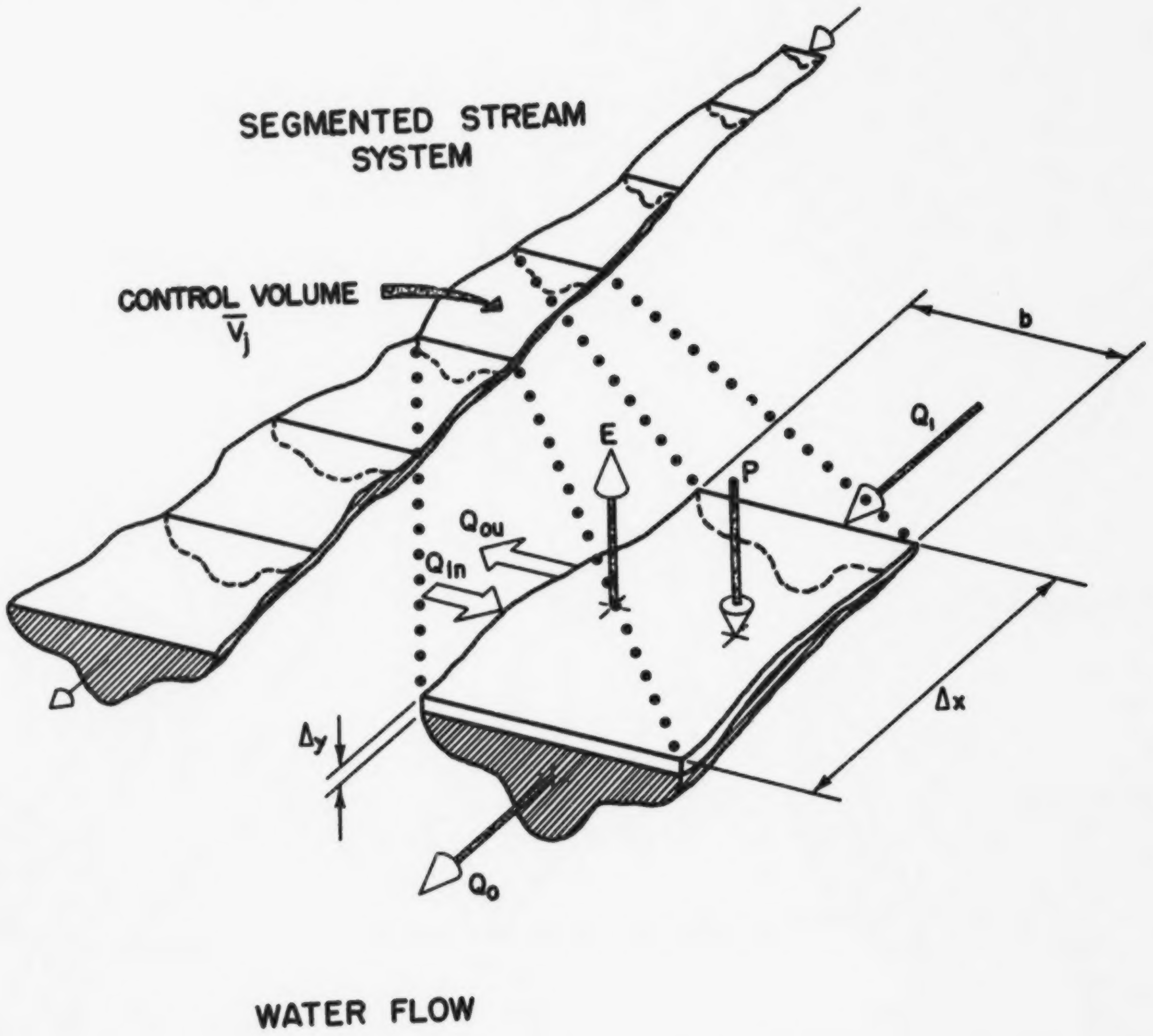
SEGMENTED STREAM SYSTEM
CONTROL VOLUME
$\overline{V}_j$
b
Q_I
E
P
Q_{ou}
Q_{in}
Δx
Δy
Q_o
WATER FLOW

FIGURE 2. CONCEPTUAL REPRESENTATION OF A VERTICALLY MIXED SYSTEM

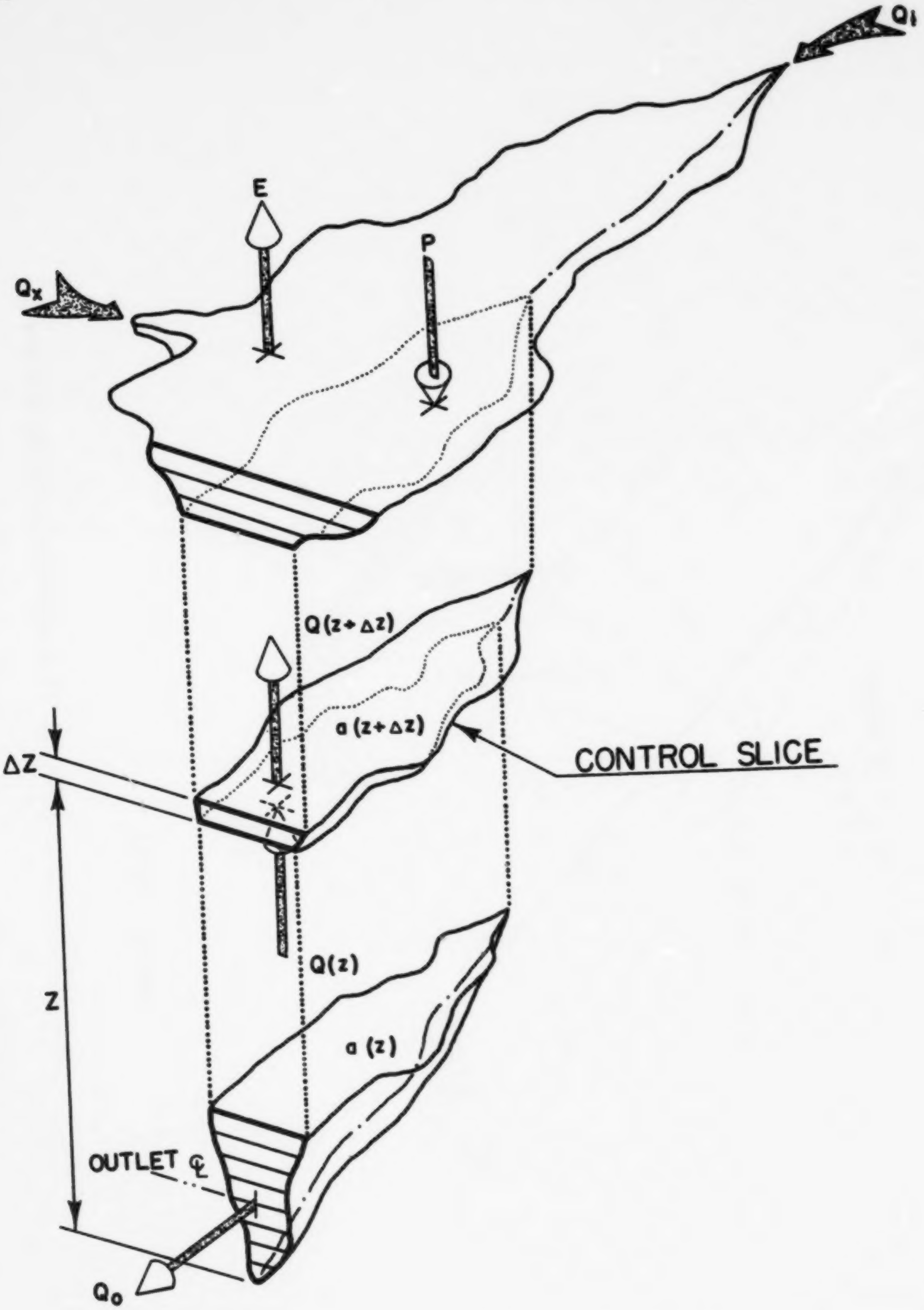
Q$_I$
E
P
Q$_x$
Q(z+Δz)
a(z+Δz)
CONTROL SLICE
ΔZ
Z
Q(z)
a(z)
OUTLET ℄
Q$_o$

FIGURE 3. CONCEPTUAL REPRESENTATION OF A RESERVOIR

motion and continuity simultaneously for discretized systems such as vertically mixed streams or estuaries (Shubinski, et. al., 1965 and FWPCA, Mar. 1966). A reservoir temperature program has also been developed to simulate the density-dependent temperature and flow conditions in the impoundment throughout an annual cycle (Orlob, et. al., 1967 and Calif. Dept. of Fish and Game, 1968). This program is capable of predicting thermal stratification and seasonal overturn, exemplified by a typical simulation shown in Figure 4.

Chemical and Biological Simulation

For a chemical or biological constituent of the ecosystem, a differential equation can be written to express the rate of change of each constituent as a function of the other variables and coefficients describing the ecologic process. For example, the biomass of phytoplankton is transported by the movement of water. In addition, phytoplankton will increase by growth depending on the light, temperature and nutrient conditions. It also decreases as result of continuous respiration, settling, and grazing by zooplankton. Terms to account for these effects can be put together to form the differential equation

$$\frac{d(\bar{V}P_i)}{dt} = T + AE\frac{dp_i}{dx} + (\mu_i - r - s)\ P_i\bar{V} - \frac{1}{Y_z}\ gp_{fi}z\bar{V}.$$

where

$\bar{V}$ = volume of the element, L^3;

P_i = mass concentration of algae in group i, M/L^3;

t = time, T;

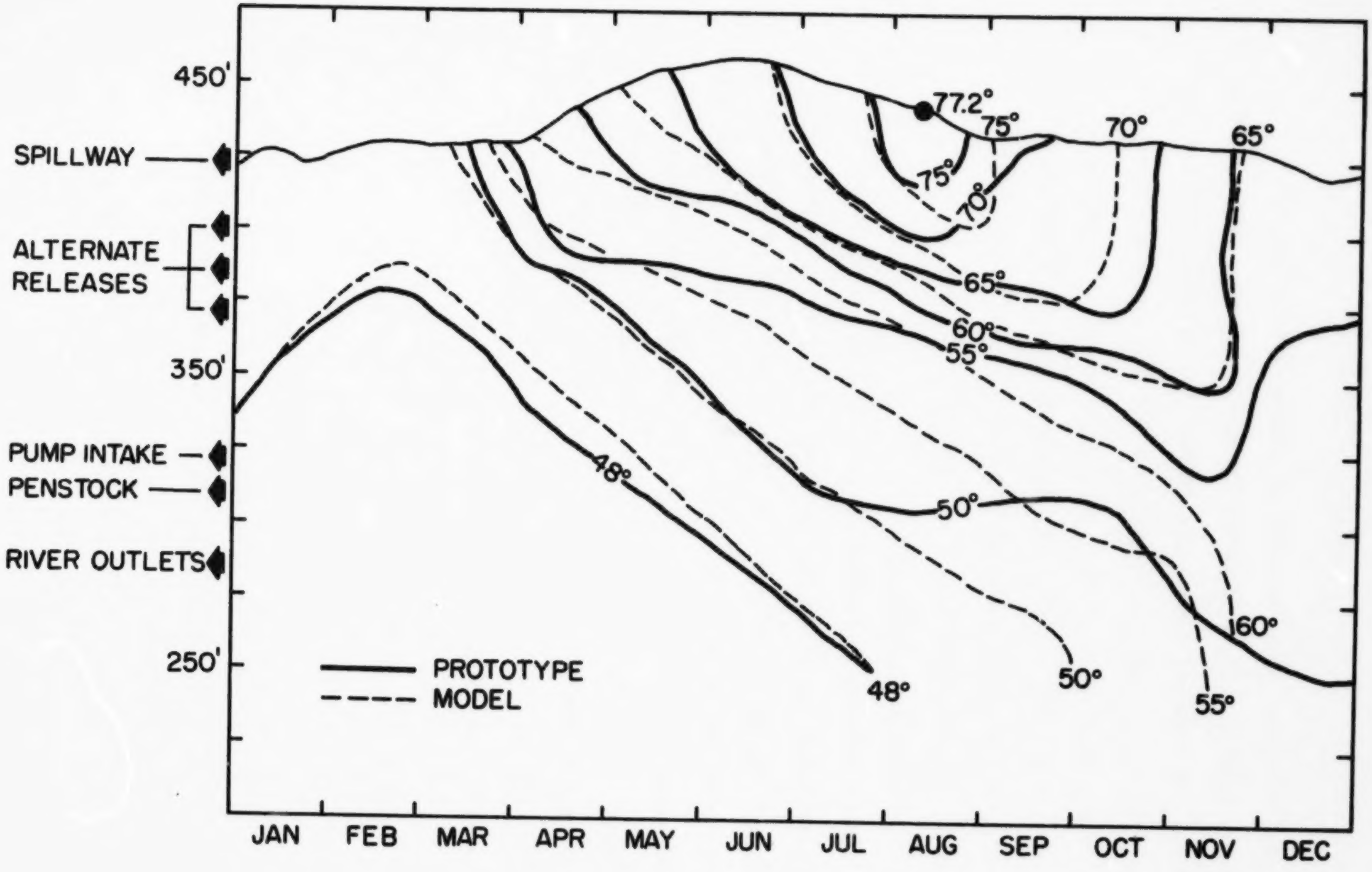

FIGURE 4. ENERGY DISTRIBUTION IN FOLSOM RESERVOIR, CALIFORNIA, 1966

T = total advective mass transfer of biomass, M/T;

A = cross sectional area of the element, L^2;

E = effective diffusion coefficient, L^2/T;

x = distance, L;

μ_i = specific growth rate of algae in group i, $^1/T$;

r = percent respired per unit time, $^1/T$;

s = percent of mass settling per unit of time, $^1/T$;

Y_z = yield coefficient of zooplankton, M/M;

g = specific growth rate of zooplankton, $^1/T$;

P_{fi} = preference factor for algae; group i, dimensionless; and

z = zooplankton biomass concentration, M/L^3.

Since the factors controlling algal growth may vary in time at various locations, a general "limiting-factor equation" has been developed for dynamic determination of μ_i, the specific growth rate of phytoplankton (FWPCA, 1968):

$$\mu_i = \hat{\mu} \left(\frac{L}{k_1 + L}\right) \left(\frac{C}{k_2 + C}\right) \left(\frac{N}{k_3 + N}\right) \left(\frac{P}{k_4 + P}\right)$$

where $\hat{\mu}$ is the maximum possible growth rate, k_1, k_2, k_3, and k_4 are coefficients and L, C, N, and P are respectively light intensity, carbon dioxide, nitrate, and phosphate concentrations. Since light intensity is nonuniform with depth, the Beer-Lambert equation was modified to treat light attenuation with depth, resulting in part from the self-shading effect of the phytoplankton. (The light intensity and the specific growth rate can be calculated dynamically for every foot and then averaged for use in the differential equation.)

In a similar fashion, a series of differential equations was formulated to describe the changing rates of zooplankton, fish, detritus, BOD, nutrients and DO. In the case of DO, terms were included to account for the decrease by BOD exertion and the increases by photosynthesis and surface reaeration. Following this approach, a dozen differential equations were written to describe the behavior of the ecosystem in each discretized element.

A computer program developed to solve the set of simultaneous differential equations will simulate the eutrophication phenomena dynamically. The interactions between biota and their environment are pursued continuously in both trophic directions, since the environmental conditions may either stimulate or inhibit the growth of biota, and the biota may both consume nutrients and/or release by-products to modify the environment. In this sense, the quality and biological relationships are seen to be "coupled", i.e., interdependent.

Functional Test of the Model

The applicability of the ecological concepts outlined above has been tested preliminarily for a simple uni-directional stream. A hypothetical stream with a mean depth of 10 feet and mean width of 500 feet was segmented into 1200-foot elements. The stream, with a flow of 150 cfs, was assumed to receive a waste discharge of 3 cfs at an intermediate element. Water quality characteristics of both the river and the waste in this example are given in Figure 5. Solar insolation was assumed to be 0.2 langleys/min at the outer atmosphere, with a cloud cover of 30 percent throughout the day. Other necessary coefficients were obtained from the literature or assumed, and the calculations were performed for every six

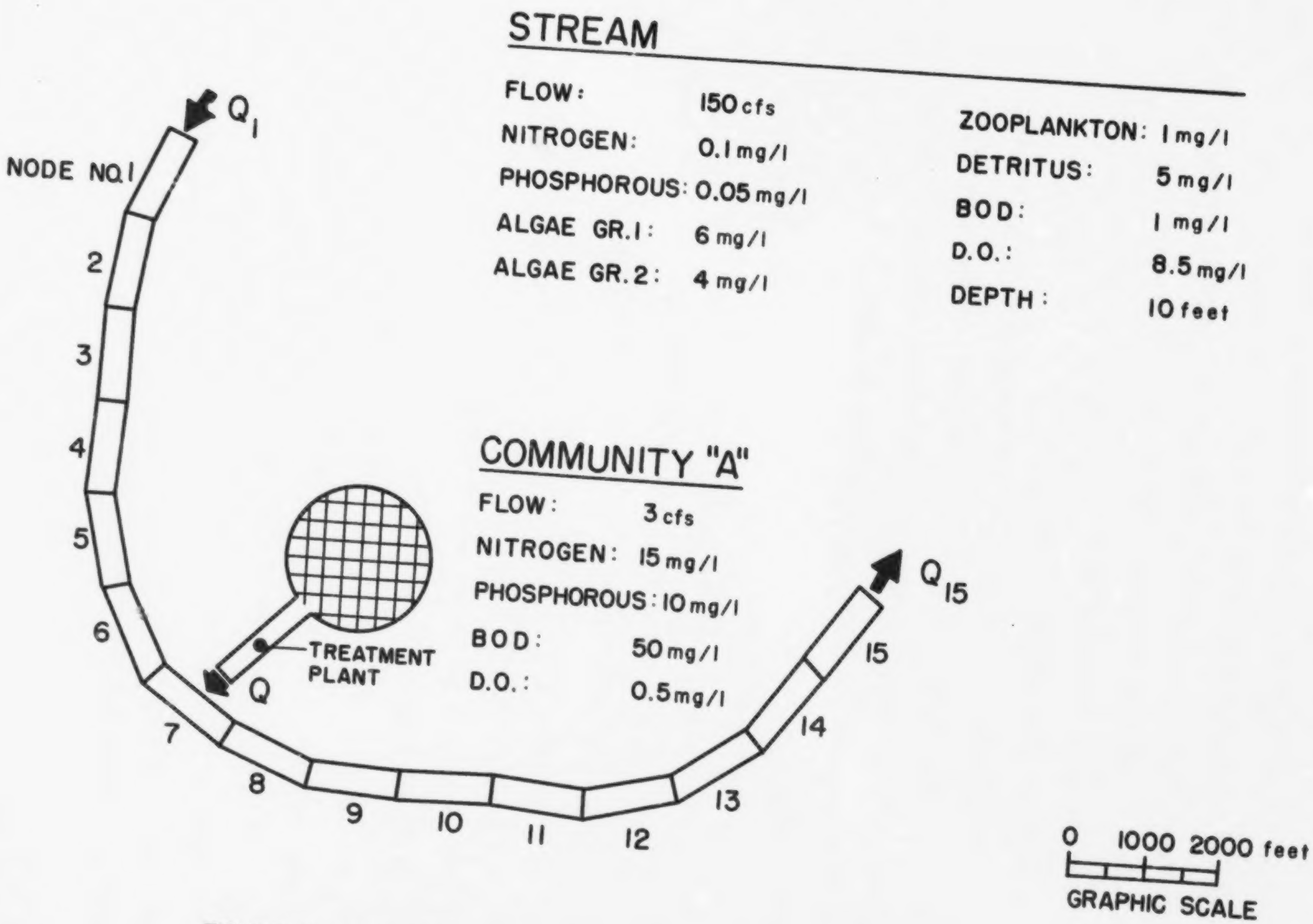
STREAM
FLOW: 150 cfs
NITROGEN: 0.1 mg/l
PHOSPHOROUS: 0.05 mg/l
ALGAE GR.1: 6 mg/l
ALGAE GR.2: 4 mg/l
ZOOPLANKTON: 1 mg/l
DETRITUS: 5 mg/l
BOD: 1 mg/l
D.O.: 8.5 mg/l
DEPTH: 10 feet
COMMUNITY "A"
FLOW: 3 cfs
NITROGEN: 15 mg/l
PHOSPHOROUS: 10 mg/l
BOD: 50 mg/l
D.O.: 0.5 mg/l
Q_1
NODE NO. 1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Q_{15}
Q
TREATMENT PLANT
0
1000
2000 feet
GRAPHIC SCALE

FIGURE 5. SEGMENTED STREAM SYSTEM FOR THE TEST CASE

hours to illustrate the diurnal variation of each parameter.

Figure 6 shows several resulting concentration profiles of important parameters, as calculated by the model. They appear to be in reasonable agreement with limited field observations in similar prototype situations. Also, detritus production showed a steady increasing rate of 0.2 mg/l per day which was consistent with the "aging" process of the ecosystem as it has been observed in the field.

The results of the simulation are indeed very preliminary and need further refinement. However, they do demonstrate the reasonableness of the technique and suggest the myriad possibilities for analysis, decision-making, and management that this developing tool provides.

The Utility of Ecologic Models

Since the ecologic model integrates all the physical, chemical, and biological factors together in one package, one obvious utility of the model is to provide a unified interpretation of data collected in the field. Aside from that, the model as described herein may serve to:

1. develop a better understanding of the behavior of the ecosystem, and
2. assist in the intelligent planning and management of water resources systems.

These two utilities actually go hand in hand, because comprehensive planning can only come from an intimate knowledge of the system's behavior.

Because the ecosystem is intricately balanced, ecologic responses are usually subtle, especially when they are attributable to several factors which may be varying either individually or jointly. Assessment of ecologic response becomes extremely difficult and has relied heavily on

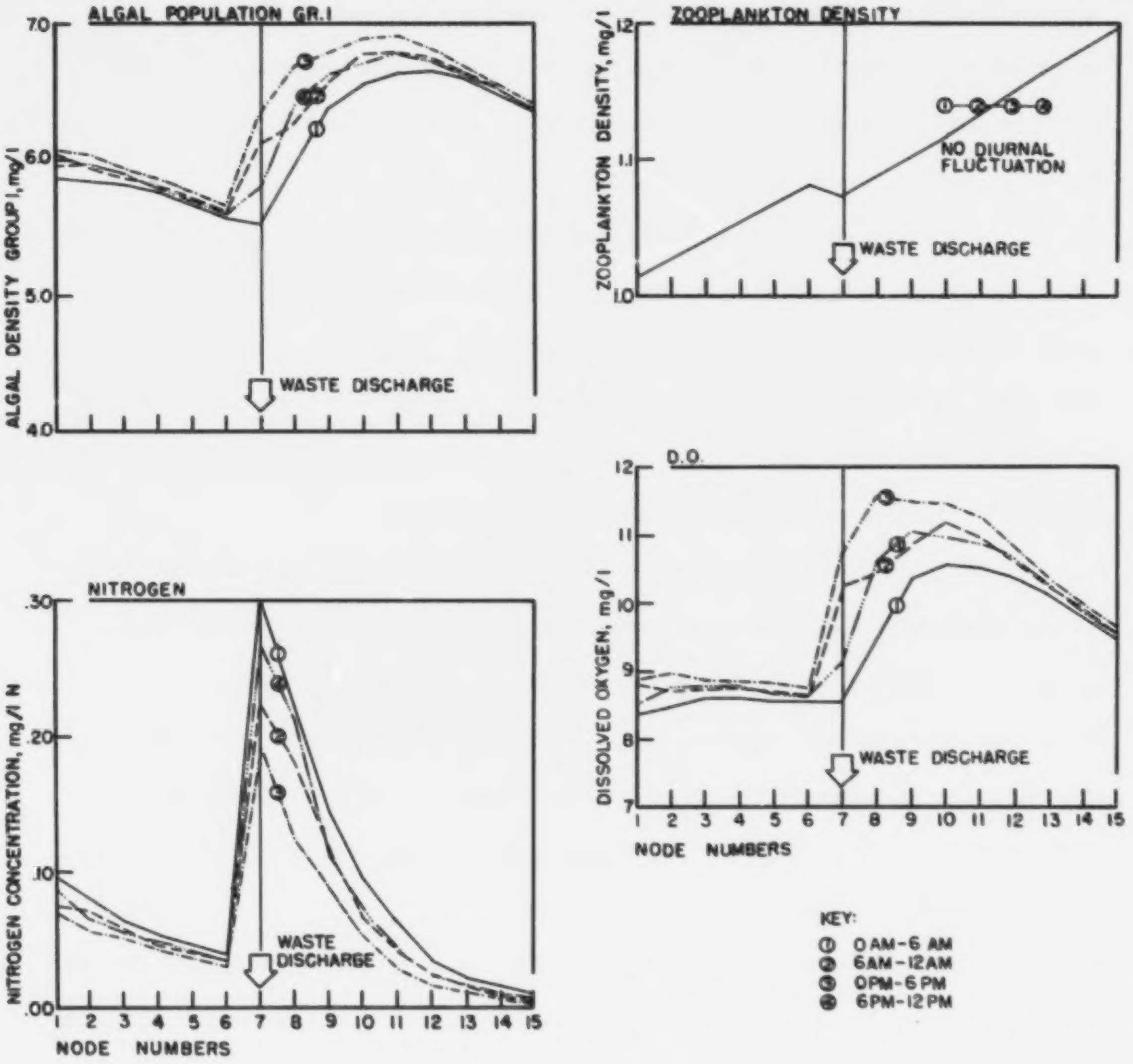

FIGURE 6. SOME TYPICAL CONCENTRATION PROFILES IN THE HYPOTHETICAL ECOSYSTEM

statistical approaches typified by multivariate analysis. However, statistical analysis, which is basically a "block-box" approach, is not particularily enlightening for depicting in a comprehensive way what is actually taking place in the system.

The model described should enable ecologists to gain an insight to the complex system, notwithstanding the simplifications and idealizations used. Intermediate results of model output may be used to evaluate both the final state and the process of ecologic responses. This in itself may stimulate scientists for further theoretical developments which are an important ingredient for the advancement of science.

Another way in which the model may contribute to the understanding of eutrophication phenomena is through sensitivity analysis. Once the accuracy of the model has been determined, it can be used to assess the effect of changing the assumed or derived rate coefficients. For example, one may wish to examine the relative importance of nutrient resolubilization from detritus. This may be accomplished by operating the model with different coefficients for detritus decay.

Figure 7 shows the responses of such a sensitivity analysis for the test case described previously. The results indicate that resolubilization of nutrients from detritus will support higher algal growth. For a thermally stratified reservoir, it may also show that the decay of detritus can deplete oxygen in the hypolimnion, allow resolubilization under the anaerobic conditions, and hence permit an algal bloom during or just after the seasonal overturn of the lake.

In a similar fashion, sensitivity analysis can be made for respiration rate, sedimentation rate, photosynthesis oxygenation factor, or the

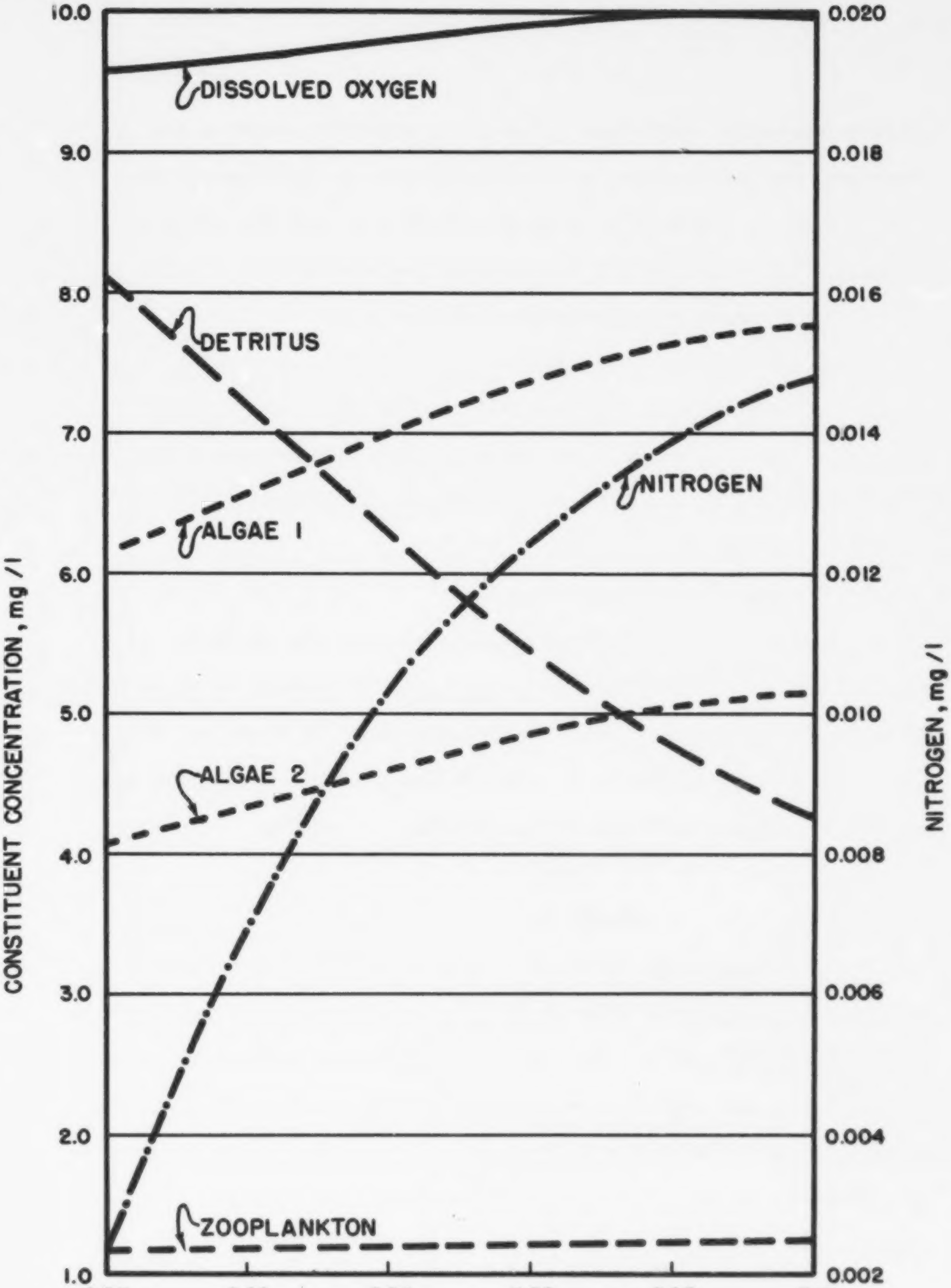

FIGURE 7. ECOLOGIC RESPONSES TO VARYING DECAY RATE OF DETRITUS (0-6 P.M., 10 TH DAY, NODE 15)

surface reaeration coefficient. Through an organized review of the responses, one may identify important parameters for additional study.

Finally, the ecologic model may serve as a tool for resources planning. The normal process of resources planning entails:

1. formulation of planning objectives,
2. design of alternative plans,
3. evaluation of the feasibility of alternatives,
4. modification of planning objectives, if required, based on the objectives' technical and economical attainability, and
5. selection of optimum plans.

Figure 8 is a schematic diagram of the planning procedure for regional water and land resources development. An ecologic model is depicted an "analytical balance" for resources planning. It weighs all the factors to ensure that the socio-economic design of the region will be compatible with the water quality and ecologic objectives.

SUMMARY AND CONCLUSIONS

An ecologic model based on fundamental principles of biology, chemistry and physics has been described. Basic ecologic processes including photosynthesis, respiration, zooplankton grazing, fish predation, sedimentation, nutrient recycling, and others were shown to be capable of representation by mathematical functions. These functions can be assembled and operated together to simulate simultaneously, the physical, chemical and biological behaviors of an ecosystem.

Preliminary tests of the ecologic model indicate the reasonableness

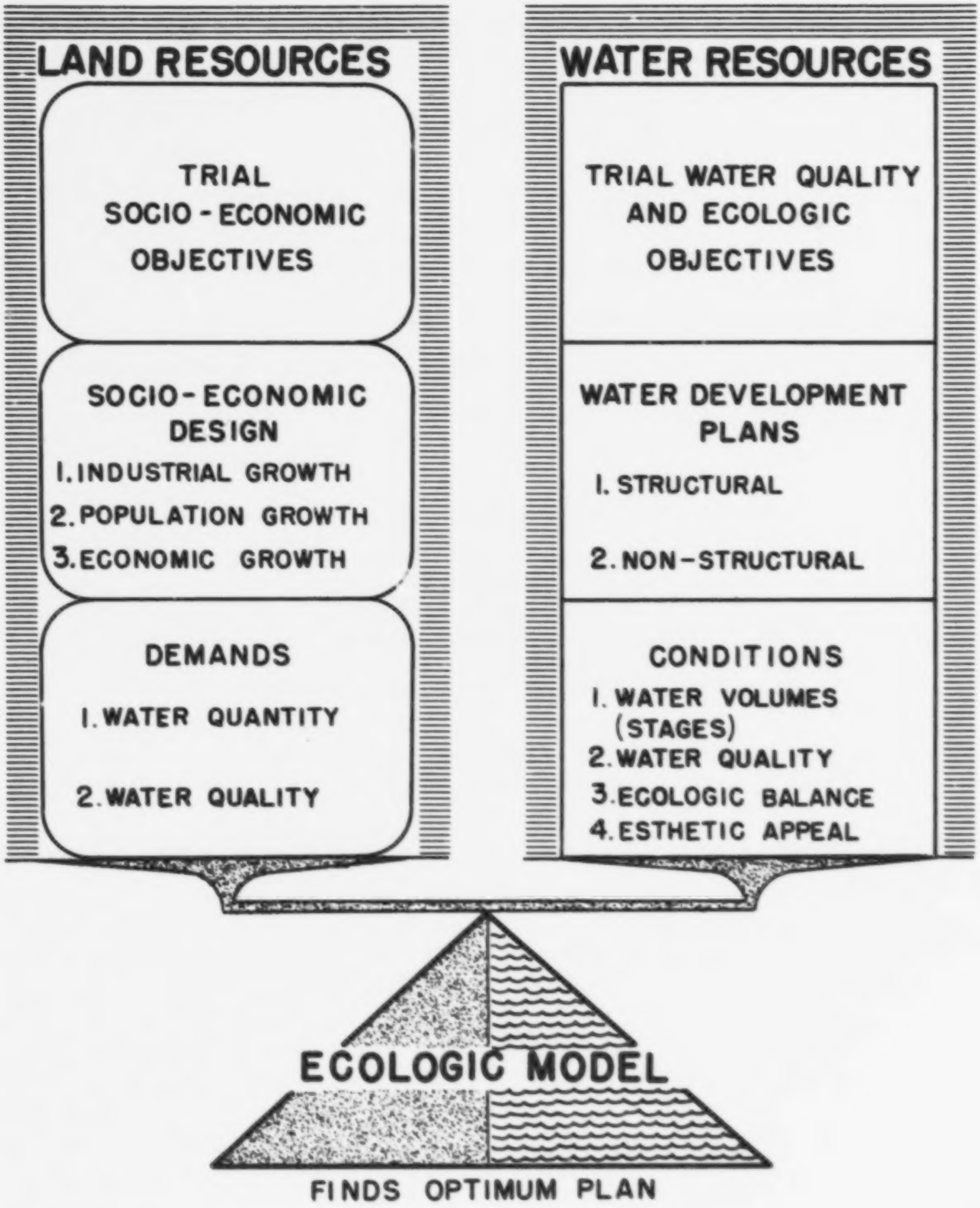
LAND RESOURCES
TRIAL SOCIO-ECONOMIC OBJECTIVES
SOCIO-ECONOMIC DESIGN
1. INDUSTRIAL GROWTH
2. POPULATION GROWTH
3. ECONOMIC GROWTH
DEMANDS
1. WATER QUANTITY
2. WATER QUALITY
WATER RESOURCES
TRIAL WATER QUALITY AND ECOLOGIC OBJECTIVES
WATER DEVELOPMENT PLANS
1. STRUCTURAL
2. NON-STRUCTURAL
CONDITIONS
1. WATER VOLUMES (STAGES)
2. WATER QUALITY
3. ECOLOGIC BALANCE
4. ESTHETIC APPEAL
ECOLOGIC MODEL
FINDS OPTIMUM PLAN

FIGURE 8. "ANALYTICAL BALANCE" FOR RESOURCES PLANNING of the technique. It is concluded that model development and application will aid greatly in the development of a more fundamental understanding of eutrophication processes and their control.

ACKNOWLEDGEMENT

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BIOGEOCHEMICAL MODELING OF EUTROPHIC LAKES

A Position Paper

by

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BIOGEOCHEMICAL MODELING OF EUTROPHIC LAKES

Great concern has been expressed regarding the present and future quality of our fresh water lakes. As a result attention is beginning to focus on developing legislation and technological methods in order to attempt to control the rapid deterioration of our lakes. If one is to accurately determine how to effectively control lake eutrophication and/or to achieve restoration, however, consideration must first be given to delineating the sources of pollution and evaluating their relative effects on the ecological system of the lake in question. Such a determination would indicate those pollution control procedures that would most significantly enhance the recovery of a lake. An analysis of this nature can be effectively achieved by utilizing computer techniques to model the biogeochemical characteristics of the lake.

The purpose of this study was to develop a mathematical description of the ecological system of an eutrophic lake such that the dynamics of this system in a changing environment can be accurately predicted and controlled. Computer techniques have been utilized in an attempt to develop a lake model which could be used to predict water quality of a lake in future years as a result of pollution and/or to establish priorities for the selection of methods of external manipulation of a eutrophic lake (e.g., pollution abatement, bottom sealing, dilution with nutrient free water, etc.) which would most significantly enhance its recovery. Since it was the purpose of this model to provide information of an overview nature concerning the future water quality of a lake, the model as such was synthesized in a general form rather than attempting to precisely discern and model each interaction. The model is intended to show general trends and for this purpose is founded, in the author's opinion, on relationships sufficiently accurate and fundamental to the intended purpose and is purposely not diluted with superfluous details. It is felt that such models could be used to generate information which may prevent future large-scale water quality management problems.

EUTROPHIC LAKE MODEL CONSIDERATIONS

The model approaches a lake as a single stage continuous flowing reactor system with specific inputs and outputs of water quantity and quality into a basin of nominally fixed volume. A hydrological balance of the inputs and outputs of water quality will establish the mean retention time (hydraulic) of a lake system while a consideration of the various water quality inputs will establish a pollutional load to the water body. The model is subsequently utilized in determining the effects of these perturbations on the fundamental microbiological and biochemical activities inherent to a lake system, which significanly affect water quality change.

HYDROLOGICAL CONSIDERATIONS

The rates of water supply and withdrawal from a lake will appreciably affect the quantity and quality of the water stored in the lake --- serving to dilute (or concentrate) the pollutants within the lake. One method of balancing the various quantities of water entering and leaving a water body during a given period stems from the classical storage equation:

$$\Delta(\text{Storage}) = \Sigma(\text{Inflows}) - \Sigma(\text{Outflows}) \qquad (1)$$

where: Δ(Storage = change in water storage volume for a given time period,

Σ(Inflows) = summation of water inflows during a given time period, and

Σ(Outflows)= summation of water outflows during a given time period.

The major components of hydrologic inflow and outflow to and from a body of water can be classified within the categories shown in Figure 1. Major souces of water inflow would include: precipitation falling directly on the water surface (P), influent tributary streams (T), treated and untreated wastewater discharges (D), runoff from storms appearing as overland flow and interflow directly to the lake (R), and seepage from the ground water reservoir (U). Major categories of water removal would include: evaporation from the water surface (E), effluent tributary streams (S), withdrawals for beneficial use (W), and seepage into the ground water reservoir (U). Using these major hydrologic components to equate the difference between the summarized inflows and outflows to change in storage over a given period of time yields an expression of the form:

$$\overline{\Delta S} = \overline{I} + \overline{R} + \overline{P} \pm \overline{U} - \overline{O} - \overline{E} \qquad (2)$$

where: $\overline{\Delta S}$ = average change of water storage (volume for the specified time period,

$\overline{I}$ = average summation of influent tributary streams (T) and treated and untreated wastewater discharges (D) for the specified time period,

$\overline{O}$ = average summation of effluent tributary streams (S) and withdrawals for beneficial use (W) for the specified time period and

$\overline{P}$, $\overline{R}$, $\overline{U}$, $\overline{E}$ = average values of the components defined above for the specified time period.

All of the parameters within this equational relationship can be measured or predicted. Equation 2 provides a relationship whereby all inflows and outflows for a lake, once quantified, may be balanced to enable the determination of one unknown. In applications of this nature, Equation 2 is most conveniently utilized to determine the direction and quantity of underground seepage (U). All other inflow and outflow components of this equation can be evaluated through a study of appropriate stream flow and climatological records. The average change in water storage ($\overline{\Delta S}$) can be determined from the changes in the water surface elevation of the lake between the beginning and end of the time period under consideration. The corresponding change in volume can be obtained through the development of an elevation-volume which can be constructed from a hydrographic map for the particular lake being studied.

Determination of the direction and quantity of underground seepage (U) as described above provides for the calculation of the mean hydraulic detention time ($\overline{HDT}$) of the lake. This mean detention time can be calculated from the following equation:

$$\overline{HDT} = \frac{\text{Average Volume for the time period}}{\Sigma \text{ Average Outflows/unit time}} \tag{3}$$

Expressed in other terms Equation 3 indicates the mean residence time of water in the lake of interest. The reciprical of this parameter ($1/\overline{HDT}$) would be equal to the dilution rate or the flow per unit volume of the lake basin.

POLLUTIONAL CONSIDERATIONS

Water has the dual dimensionality of quantity and quality which together serve to define the state of water in any ecosystem. These two characteristics can be regarded as defining the components of a vector. Thus the input to and outputs from the lake, initially defined in Figure 1, are regarded as pollution vectors. The quantity and quality of the water represented by any of the vectors is used to define each of the components, e.g., runoff to the lake becomes R (Q, C_x, C_y, C_z) where Q is the quantity of water, and C_x, C_y, and C_z are the concentrations of pollutants x, y, z in the water. In this consideration the significant components of all pollution vectors must be defined and quantified before any ensuing analysis of the lake may be made.

In the strictest sense, any physical, chemical, or biological substance that is allowed to enter a lake and upset the normal aquatic balance must be considered as a pollutant to that body of water. Yet rather than to attempt to develop a model which considers all known pollutants to the lake in question, it is most efficient to select those pollutants, which because of their concentration and/or composition, most critically affect the present and/or future water quality of the lake under consideration.

In general the pollutants which are allowed to enter a lake, stimulate the growth of two broad groups of organisms: 1.) heterotrophic organisms (animals, bacteria, fungi); and 2.) autotrophic organisms (rooted or floating

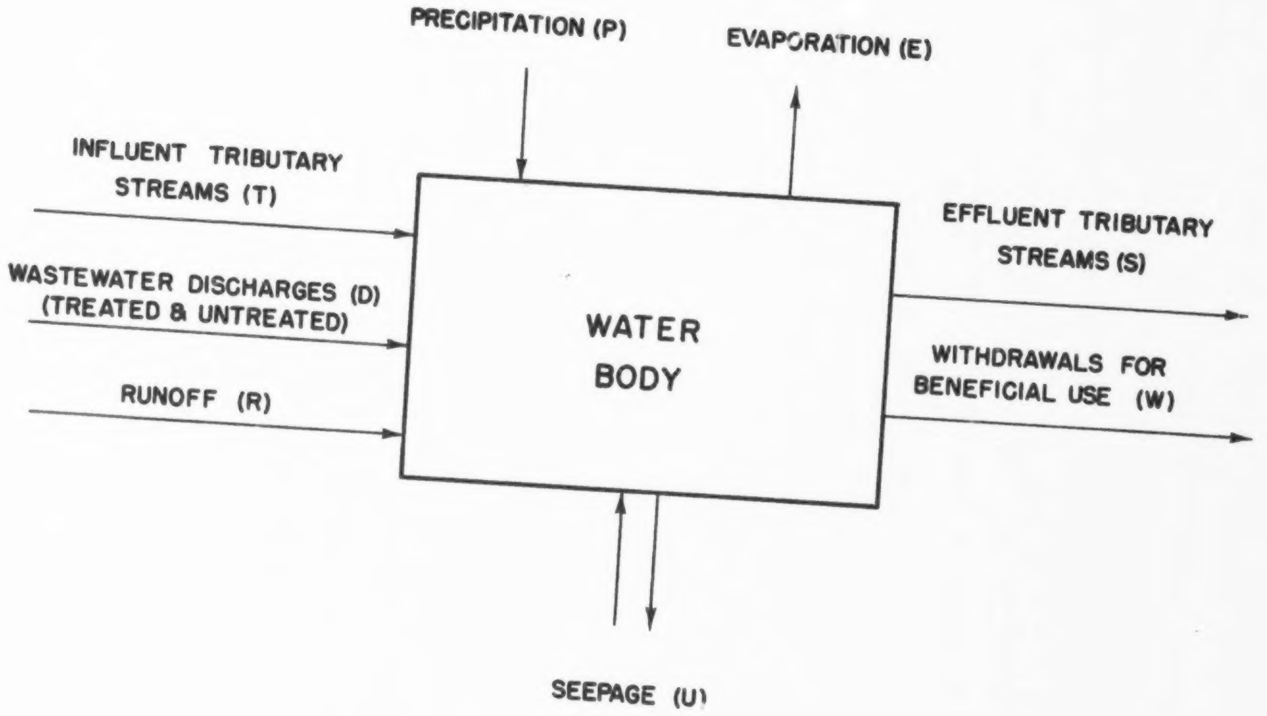

PRECIPITATION (P)
EVAPORATION (E)
INFLUENT TRIBUTARY STREAMS (T)
WASTEWATER DISCHARGES (D) (TREATED & UNTREATED)
RUNOFF (R)
WATER BODY
EFFLUENT TRIBUTARY STREAMS (S)
WITHDRAWALS FOR BENEFICIAL USE (W)
SEEPAGE (U)

FIGURE I. MAJOR CATEGORIES OF HYDROLOGIC INFLOW AND OUTFLOW TO AND FROM A WATER BODY

plants and algae). Organic carbon is generally considered as the critical element (or pollutant) necessary for the production of heterotrophic organisms with phosphorus being normally considered as the critical element for the growth of autotrophic organisms --- other environmental factors requisite to the growth of these organisms are usually unlimiting in lake systems. In most cases, therefore, the concentrations of organic carbon and phosphorus determine the extent of heterotrophic and autotrophic activity in a lake and thus usually become the key pollutants to label. As a result of this heterotrophic-autotrophic activity a third pollutant, the concentration of suspended solids is usually of equal significance as this parameter will reflect the newly synthesized biomass in the lake. Thus, in the type of lake model being considered, the principal pollutants of concern are: organic material (C_c), phosphorus (C_p), and suspended material (dry weight at 103° C; C_{ss}).

The concentrations of these pollutants in the hydrological inflows to a lake basin can be most accurately determined by direct measurement over the time period of interest. The concentration of some of these pollutants can vary appreciably within the input classifications shown in Figure 1. For illustration, Table 1 indicates ranges for these pollutants as reported by various literature sources for: treated and untreated domestic wastewaters, urban and rural runoff, precipitation and groundwater seepage. This table has been developed for guidance and is not intended to be complete.

TABLE I

Typical Ranges of Suspended Material, Organic Material and Phosphate Concentrations in Various Hydrological Lake Inputs.*

Input Source	Suspended Solids (mg/l - dry wt.)	Organic Material (mg/l as COD)	Ortho-Phosphate (mg/l as P)
Domestic Wastewater (D)			
1. Untreated	200 - 1000	100 - 400	1.0 - 10.0
2. Primary	20 - 200	70 - 300	0.5 - 9.0
3. Secondary	5 - 200	10 - 100	0.1 - 7.5
4. Tertiary	5 - 50	0.1 - 10	0.03 - 0.30
Runoff (R)			
1. Urban	5 - 1200	20 - 600	0.1 - 0.5
2. Rural	5 - 2000	30 - 300	0.3 - 5.0
Precipitation (P)	2 - 35	5 - 50	0.01 - 0.10
Seepage (U)	20 - 1500	5 - 500	0.01 - 1.0

*Ranges established with guidance from the following references: (1) (2) (3) (4) (5) (6).

INTERNAL RELATIONSHIPS OF THE LAKE MODEL

The internal relationships occurring within a lake are numerous and frequently extremely complex; in addition, many of these relationships are not clearly understood at this time. Broadly speaking, microbial, chemical, and geological reactions are all inherent to such a system and all will be affected by the hydrologic and pollutional inputs/outputs previously cited. Developing a model which attempts to incorporate all known interactions, however, not only is a herculean task, but would be an atrocious waste of time as any subsequent sensitivity analysis would clearly show that most of the interactions are negligible in terms of their importance to establishing the present or future state of the entire ecosystem. Obviously, attention must be given to accurately modeling the fundamental or controlling relationships of a lake, to a degree sufficient for the intended purpose of the model. The goal in this case, as mentioned previously, being to develop a model which would be suitable for accurate prediction of the future water quality of a lake as a result of pollution and/or to yield data sufficient for the assignment of priorities in the selection of methods of lake restoration.

As indicated previously organic carbon and inorganic phosphorus are frequently considered as the critical water quality parameters (pollutants) necessary for energizing the heterotrophic and autotrophic cycles, respectively, within aquatic environments. The continuous movement of these quality parameters within the ecosystem is represented by the model in Figure 2. Specific application of this ecosystem model to a lake environment is pictured in Figure 3. Organic material entering a body of water serves as a source of energy for heterotrophic organisms in the synthesis of new

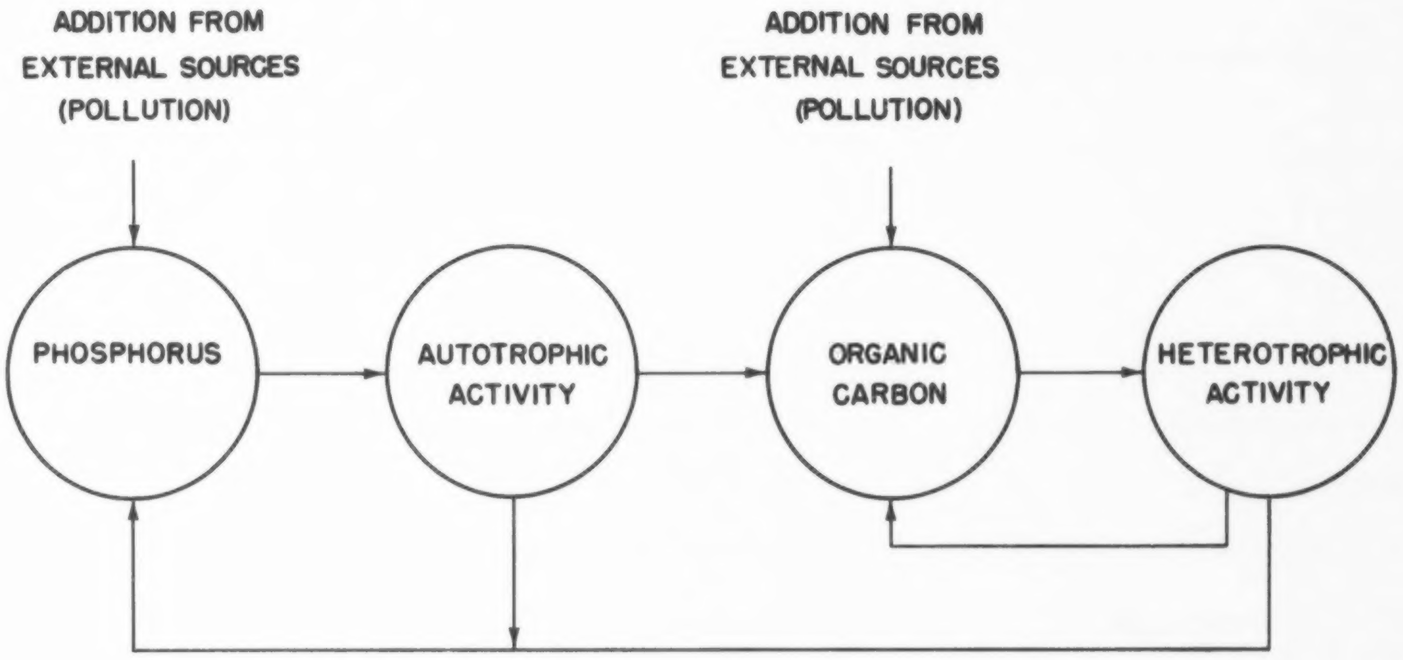

ADDITION FROM
EXTERNAL SOURCES
(POLLUTION)
ADDITION FROM
EXTERNAL SOURCES
(POLLUTION)
PHOSPHORUS
AUTOTROPHIC
ACTIVITY
ORGANIC
CARBON
HETEROTROPHIC
ACTIVITY

FIGURE 2. CARBON AND PHOSPHORUS CYCLING IN NATURAL ECOSYSTEMS

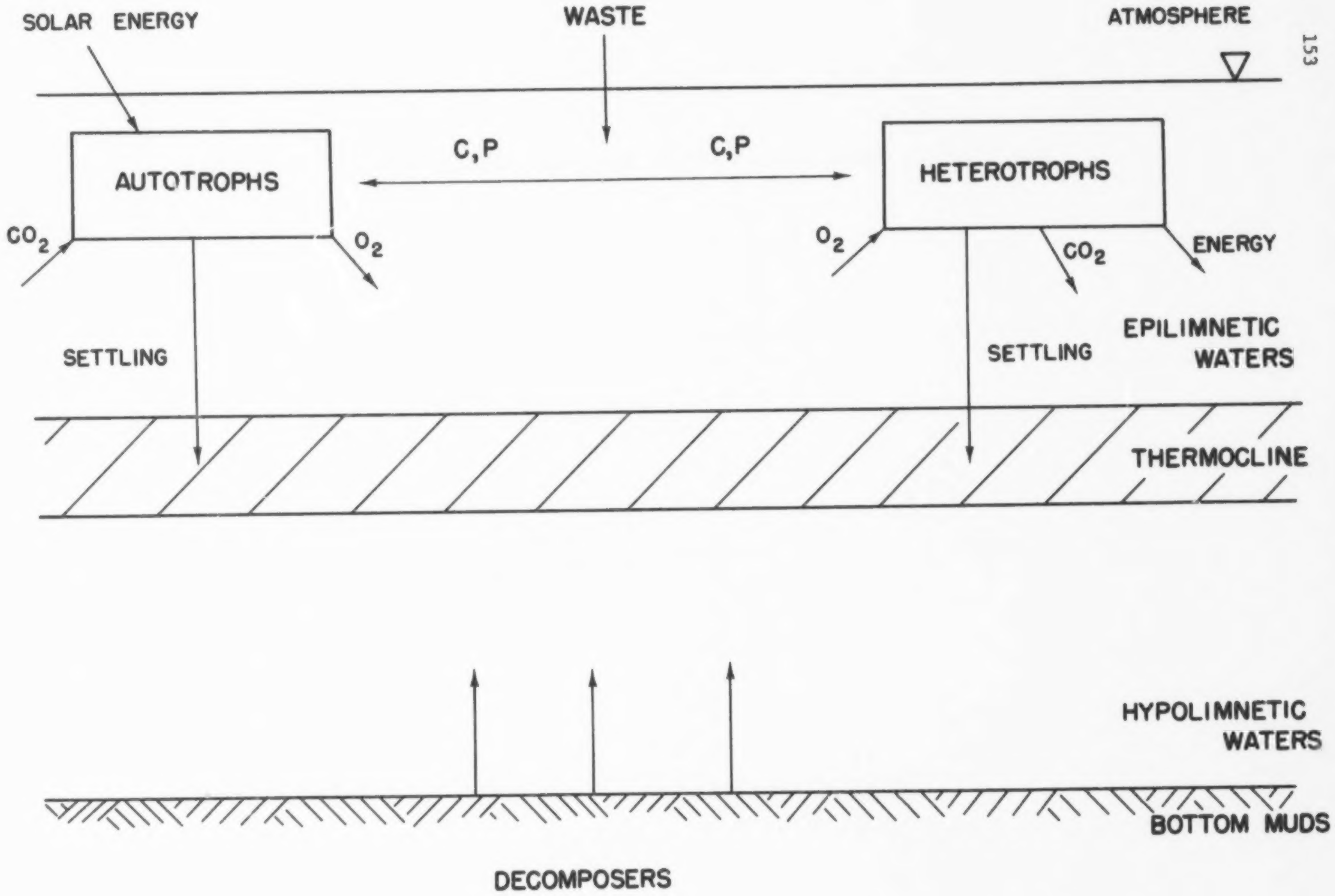

SOLAR ENERGY
WASTE
ATMOSPHERE
C,P
C,P
AUTOTROPHS
HETEROTROPHS
CO2
O2
O2
CO2
ENERGY
SETTLING
SETTLING
EPILIMNETIC WATERS
THERMOCLINE
HYPOLIMNETIC WATERS
BOTTOM MUDS
DECOMPOSERS

FIGURE 3. BIOGEOCHEMICAL EXCHANGES IN AQUATIC ECOSYSTEMS

cellular protoplasm, which subsequently results in the release of carbon, nitrogen and phosphorus into the water as soluble pollutants. This portion of the total biosystem within an eutrophic lake is represented by the following relationship:

$$C_a\ H_b\ N_c\ O_d\ P_e + (a + 1/4\ b - 1/2\ d + 3/2\ c + 2\ e\ )\ O_2 \rightarrow A\ CO_2$$

$$+ b/2\ H_2O + c\ NO_3^- + e\ PO_4^{-3} + C_{106}\ H_{180}\ O_{45}\ N_{16}\ P_1 \qquad (4)$$

Autotrophic organisms capture solar energy and utilize carbon dioxide and other inorganic species in the synthesis of new algal protoplasm in accord with the following reaction:

$$106\ CO_2 + 90\ H_2O + 16\ NO_3^- + 1\ PO_4^{-3} + \text{Solar Energy} \rightarrow$$

$$C_{106}\ H_{180}\ O_{45}\ N_{16}\ P_1 + 154\ 1/2\ O_2 \qquad (5)$$

These newly synthesized autotrophic and heterotrophic organisms subsequently settle to the lake bottom and are subjected to anaerobic decomposition as generally represented by:

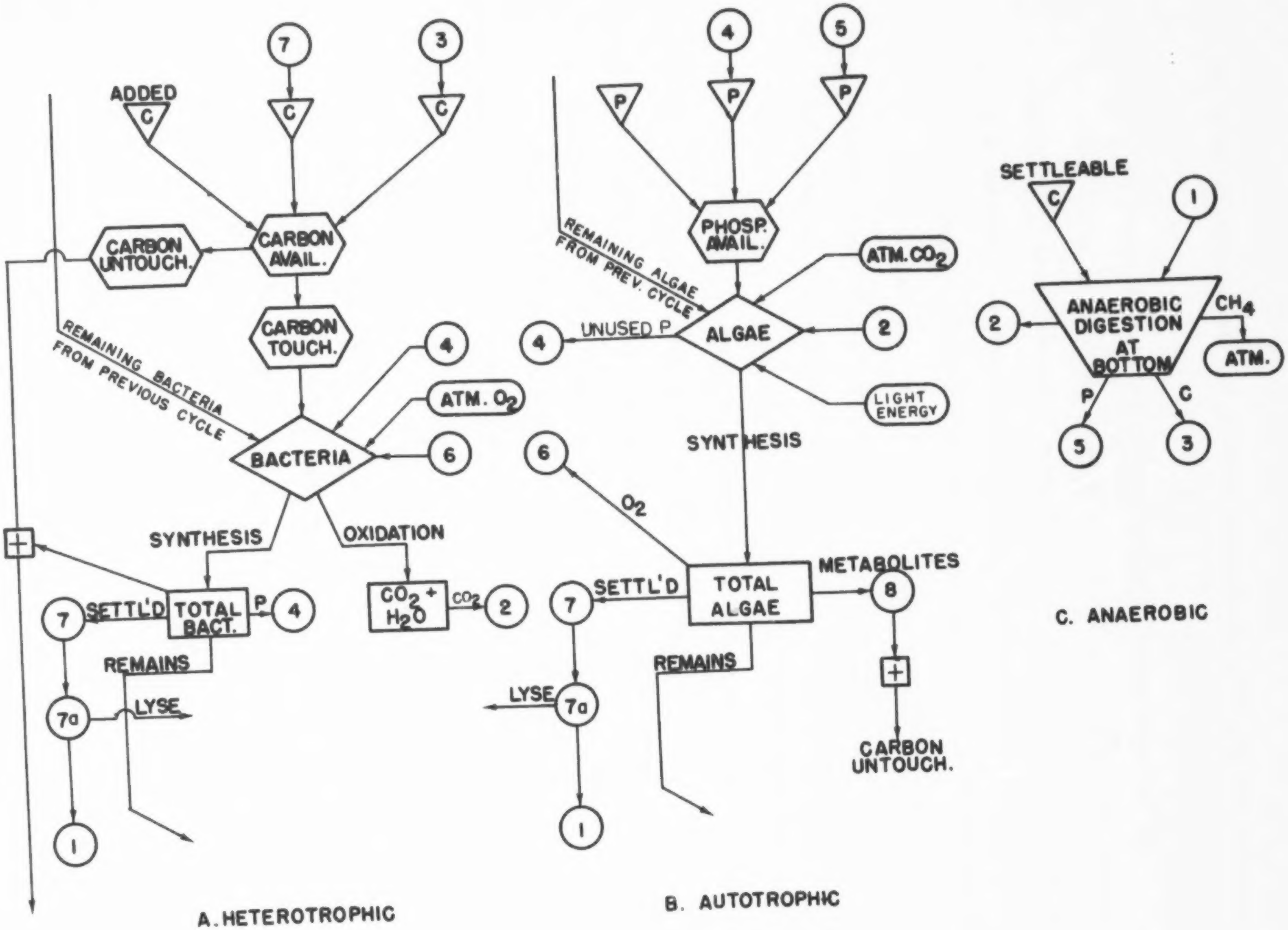
$$C\ H\ O\ N\ S\ P \rightarrow R\ C\ O\ O\ H \rightarrow CO_2 + CH_4 + NH_4 + H_2S + PO_4^{-3}$$

$$+ H_2O + \text{Synthesized Bacteria} \qquad (6)$$

thus releasing to the overlying water, soluble organic and inorganic materials for reutilization. The concurrent functioning of the heterotrophic, autotrophic and decomposer microbial activities represents the total biosystem that is principally responsible for the exchange of nutrients within eutrophic lakes.

Specific representation of the above equations in an eutrophic lake system can be modeled in the manner indicated in Figure 4. This Figure represents in flow sheet form the relation occurring within each subbiosystem and the interrelationships between the three biosystems. Interchanges between subsystems are defined on the flow chart (Figure 4) by circles which contain a number, the number being the label of the interchange point. Table 2 contains a description of each of the interchange points; for example, interchange point ④ is the net production of phosphorus by heterotrophs and unused phosphorus by autotrophs. Interchange between each subsystem and its environment is labeled directly, for example, externally added carbon and light energy.

Considering the heterotrophic and autotrophic systems as if they were single bacterial and algal organisms within the system, one can proceed through the cycles modeled in Figure 4. As shown in Figure 4A, the heterotrophic system, the bacterial organism encounters a certain quantity of available carbonanceous material, a fraction of which is undegradable (untouchable). That fraction which is degradable is combined with inorganic phosphorus and oxygen - ④ ⑥ atm - to produce carbon dioxide and water - ② and synthesize new organisms according to Equation 4. A net production of phosphorus is encountered because the organism releases the phosphorus contained in the carbonaceous material which it degrades - ④ and does not synthesize this phosphorus into cellular photoplasm. In the process of


ADDED
C
7
3
CARBON AVAIL.
CARBON UNTOUCH.
CARBON TOUCH.
REMAINING BACTERIA FROM PREVIOUS CYCLE
4
ATM. O_2
6
BACTERIA
SYNTHESIS
OXIDATION
TOTAL BACT.
SETTL'D
P
CO_2 + H_2O
CO_2
2
REMAINS
7a
LYSE
1
+
5
P
PHOSP. AVAIL.
REMAINING ALGAE FROM PREV. CYCLE
ATM. CO_2
ALGAE
UNUSED P
LIGHT ENERGY
SYNTHESIS
O_2
TOTAL ALGAE
METABOLITES
8
CARBON UNTOUCH.
SETTLEABLE
ANAEROBIC DIGESTION AT BOTTOM
CH_4
ATM.

A. HETEROTROPHIC

B. AUTOTROPHIC

C. ANAEROBIC

FIGURE 4. FLOW CHART OF INTERCHANGES BETWEEN BIOLOGICAL SUBSYSTEMS

oxidation and synthesis the organism produces certain metabolites which are assumed to be nondegradable.

Of the total number of organisms present, a certain fraction settles out - ⑦ ; the fraction remaining continues on to the next cycle. Of the quantity of settled bacteria a fraction lyses (7a) introducing a quantity of degradable organic material that is considered an input to the next cycle. The remainder of the settled bacteria which did not lyse continues to the anaerobic system at the bottom of the lake - ① .

Referring to Figure 4B, the autotrophic system, the algal organism encounters a certain amount of available phosphorus to which it adds carbon dioxide and solar energy to synthesize new algal cells in accordance with Equation 5. In the process of photosynthesis algal cells produce oxygen - ⑥ - and cellular metabolites - ⑧ ; the fraction of algae that remains suspended continues on to the next cycle. As with the heterotrophic system a certain fraction of the algae settles out - ⑦ - and a certain fraction of these lyse and introduce degradable organic material to the heterotrophic system (7a) . The remainder of the settled algae continue to the bottom anaerobic system of the lake ① .

Figure 4C models the anaerobic system in the bottom muds. The anaerobic system receives inputs from the heterotrophic and autotrophic systems ① and from external sources as settleable organic solids. The process of anaerobic decomposition produces methane and carbon dioxide ② ; also according to Equation 6 degradable organics ③ and inorganic phosphorus ⑤ are released into the lake system.

TABLE 2

Description of Flow Chart Interchange Points

Interchange Point	Description
①	Settled algae and bacteria to the bottom muds.
②	Carbon dioxide produced by the heterotrophs and utilized by the autotrophs.
③	Organic carbon released by the anaerobic system.
④	Net production of phosphorus by heterotrophs and unused phosphorus by autotrophs.
⑤	Phosphorus released by anaerobic system
⑥	Oxygen produced by photosynthesis.
⑦	Quantity of heterotrophs and autotrophs which settle.
(7a)	Fraction of ⑦ which lyse to produce (① = ⑦ - (7a))available organics
⑧	Undegradable metabolites of the autotrophs.

SPECIFIC EUTROPHIC LAKE SIMULATION

Simulation of a lake's development is the process of operation of the model of the lake system. The vectors of Equation 2 define the hydrologic and quality inputs to the biological model of the lake. The biological model of Figure 4 operates from one cycle to another as indicated in the Figure and as described in the above section. The quality parameters of the vectors translate to quantities of carbon, phosphorus, and suspended solids. Each of these quantities is input to the biological subsystems of Figure 4. These subsystems transform the quantities of carbon, phosphorus, and suspended solids into new microorganisms and by products. At this point new quantities of quality factors are introduced to the lake, and combined with the by products of the above process are introduced to the biological subsystems. In this way, a continuous operation of the lake system is maintained. By performing this simulation on a digital computer, the lake may be allowed to operate for many years in a matter of a few minutes. If one records the parameters that determine the state of the lake quality throughout the simulation, a history of the lake is developed that spans from the point in time when sufficient data are known to a point where the state of the lake's quality is of interest.

The natural overturn of the lake provides a basis for defining the time scale of the simulation. This turnover occurs in the Spring and Fall seasons of the year. These two major events serve to break the simulation into successive time periods of approximately six months. After Fall overturn, the entire lake is considered to be a completely mixed system which operates under steady states conditions. The lake operates in this manner until the Spring when overturn again occurs. During this subsequent new period the lake is considered to be stratified into two regions. The upper region consists of the top 2 meters of the lake while the remainder of the lake constitutes the lower region. Each region of stratification is considered to be completely mixed and to be operating under steady state conditions.

Since the biological system is considered to be in a steady state, the heterotrophic system, for example, may be considered to synthesize a quantity of new microorganisms that is a fixed fraction of the amount of metabolized organic material. By the same reasoning the quantity of oxidation products produced is also a fixed fraction of the amount of degradable organic material being metabolized.

A similar situation occurs in the case of the autotrophic system. In this case the availability of inorganic phosphorus is assumed to be the critical component of algal synthesis, and hence, the quantity of new algal cells produced during any cycle is determined by a full utilization of all available inorganic phosphorus. The anaerobic system at the bottom of the lake operates in a similar manner to the heterotrophic system. A fixed fraction of the available organic material is considered to be released and subsequently become available to heterotrophic decomposition as volatile acids, while the remainder of the volatile acids produced with this material are further decomposed by the anaerobic system to methane and carbon dioxide. Since the lake is in a steady state condition the fraction of the total number of microorganisms in the heterotrophic system and autotrophic system that settle out is considered a constant from cycle to cycle. By the same reasoning the fraction of settled organisms that lyse is considered constant.

In order to stimulate the lake system using the model proposed in this section, the parameters involved in the model need not be known *a priori*.

It is possible to derive from the literature and from experimentation, estimates of probable parameter values or ranges within which it is felt they should lie (See Table 1). Such estimates are considered as initial conditions for this simulation. Operational values under consideration in this study are shown in Table 3. The upper portion of this Table considers the operational values (percentages) for the flow of carbon through the simulated eutrophic lake system. In the bottom section of Table 3, the fractional distribution of surface runoff from different types of land areas is indicated,with the fractional composition of carbonaceous material in the runoff also being shown. Because one can measure the current state of the lake in terms of the objectives of the simulation, values for the important parameters may be adjusted by simulation to obtain the desired objectives.

The above described eutrophic lake simulation procedure has been applied to a test lake system (Stone Lake, Cassopolis, Michigan) for the purpose of establishing the eutrophication development in this specific lake example. This modeling approach was used to reflect back in time and synthesize the pollution history of Stone Lake from year 1939 to 1965 at which time direct domestic pollution of the lake was terminated. Particular emphasis has been given to the water quality parameters of soluble organic carbon, inorganic phosphorus and dry weight of suspended microorganisms. Shown in Table 4 are water quality data revealing the pollutional development in Stone Lake which has produced intensive symptoms of eutrophication. The data in this Table were generated by the lake simulation model through numerous programing trials and operational parameter adjustments. Simulation values for 1965 agree quite closely with those measured in the lake at that

TABLE 3

Operational Values for Eutrophic Lake Simulation

Initial Parameter Values
(expressed as percentage)

		Range
**Untouchable Carbon Fraction	25.00	25.0>UTC>10.0
**Percent Carbon Synthesized	15.00	10.0< P <30.0
Bottom Release of Carbon	30.00	
Heterotroph Fraction Settled	4.40	
Autotroph Fraction Settled	4.40	
Heterotroph Fraction Lysed	10.00	
Autrotroph Fraction Lysed	10.00	
Carbon Available - Metabolites	1.00	
Autotroph Metabolites	0.00	
Auto. Metabolites Available	0.00	
Lag Time for Bottom Release	0	

Breakdown of Runoff Volume
(expressed as percentage)

Fraction of Runoff from	Carbonaceous Material	
	Suspended	Untouchable
Forrested 19.50	25.00	10.00
Urban 75.00	25.00	10.00
Agriculture 5.50	25.00	10.00

TABLE 4

Changes in Lake Water Quality Parameters
(Values expressed in mg/l)

Year	Soluble Organic Material (as COD)	Suspended Microorganisms		Inorganic Phosphorus (as orthophosphate)
		Heterotrophs (as COD)	Autotrophs (as COD)	
1939	1.60	0.089	1.17	0.085
1949	*(9.5) 12.2	.540	4.23	1.75
1959	*(22) 38.2	1.24	9.08	4.27
1965	*(35.0) 66.0	1.91	14.80	6.30

*(Corrected total COD based upon known value in 1965 of $\approx$ 35.0 mg/l.

time. As shown in the Table an appropriate correction to the simulation COD value was made based upon the known value for the lake water in 1965.

SUMMARY AND CONCLUSIONS

A mathematical simulation (time discrete linear model) of the biogeochemical exchanges within a lake system has been developed; and its use in the synthesizing of the eutrophication history of a specific test lake, as the result of pollutional inputs, has been demonstrated. Since this simulation model describes, with a reasonable degree of accuracy, the progress of water quality change in an actual lake system, one can surmise that the model closely approximates the biogeochemical processes in Stone Lake and can be used to predict future water quality change as a result of certain perturbations on the lake system. The use of the model to suggest certain external manipulation procedures for the lake to enhance water quality improvement is also a true possibility.

Following this initial eutrophic lake modeling development, current and planned studies are concerned with a non-linear model concept for more realistic description of the internal lake processes involved and hence better predictive capabilities. With the non-linear modeling approach, one can determine the important ecological elements and transfer functions in the total system to a higher degree of confidence. In addition a more accurate extension of the model into the future can be made for the purpose of determining the relative significance that certain lake manipulation procedures may have on future water quality. It is strongly felt that ecological modeling developments, such as this, will greatly assist the political decision making which will lead to the development of successful concepts in water quality management.

ACKNOWLEDGEMENTS

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DISCUSSION

BIOGEOCHEMICAL MODELING OF EUTROPHIC LAKES

BELLA: I have run some simulation models and have found out that vertical mixing is very important, not just for re-cycling the nutrients, but apart from that, just from the types of algae that can, or cannot, survive. This, combined with sinking velocities, you find that the sinking velocities are higher than the growth rates, so obviously they are not going to make it. If you increase the mixing, however, this brings the algae back up and gives them a second chance. With mixing alone, in a very simplified model, the changes in mixing that occurred over a summer period have roughly corresponded to some of the observed changes. Slight changes in mixing, that's all. As far as artificial mixing, I have also simulated this, and I won't go into the details, but, in artificial mixing I think here is one of the cases where we might make a few wrong assumptions not looking into the limitations of our models. I ran artificial models and I had three different algae: a, b, and c, and the mixing seemed to indicate that the three were more compatible with the artificial mixing than without it. As we increase the mixing we reduce the diversification of the environment. We make a homogenous environment both in time and space. This reduces the number of niches into which individual species can do their thing and effectively compete with others, but we won't see this in our models because we don't have the individual species. So we've got to watch the limitations of our model, and I've used the sample of vertical mixing as an example. The model may very well show that the vertical mixing has some very good benefits, but because we don't have the diversity of the real life situation in the model, we're liable to end up with more problems than we have solved, in that a species diversity, actually the number of species, may actually decline because we have destroyed the variations within the environment.

SANDERS: I want to bring up a point that is quite important. We have found in working with the batch bacteria, that the velocity for the transport of subterranean nutrients to the surface (and this could be sediment, or a rock in a stream, or whatever) will be a governing factor in population density of organisms in this film. And also, as we carry this back to the point where we have practically no velocity or just a diffusion velocity of materials to this film, we may actually not even have a film at all, but we would have a surface with the organisms down into the model. But if we have any appreciable flow across this surface we are getting a removal of the nutrients and substrata from the overlying water, rather than those down in the mud. This is a very important factor in assessing the input from the bottom of our model or from our streams. Another factor in looking at this bacterial film, say 250 microns thick, with a micro dissolved oxygen probe, though, we found that when we measured down within 25 microns of the surface, at a flow of liquid across the film of a foot and a half per second, the variation of oxygen alone at this level varied 8/10 of a part per million. In other words, we had a fluctuation of oxygen in the water at this level above the film, such that there was a demand in the organism to move downstream and to struggle to continue to flow along the surface, so that we had fluctuation for the maximum variation here of 8/10 of a part per million. This we were able to measure. So velocity and matter transport is an extremely important factor in our systems, and one which should not be overlooked.

ECHELBERGER: When we were trying to home in on the existing water quality in this lake system, we would have difficulty making the adjustments within the internal model of various transfer functions, and settling of particulate matter was one of the things we had some difficulty with. That first few times around we filled the lake with bacteria.

BELLA: I think you can demonstrate this by the process of putting a cylinder down into the water, with the same water inside and out and not allowing any mixing. The population changes inside the cylinder as compared to outside is very different, and I think this is principally due to the mixing. Now if we wait long enough, other things will change.

SHAPIRO: This has nothing to do with mixing, but I think that just in looking at Wayne's diagram, and so on, and acting just out of sheer ignorance, Is there any possibility of approaching this problem with electrical analogue procedures? I have been very impressed with what hydraulics people and limnologists can do with analogs. Is this a feasible approach? I am talking about an electrical analog rig, in which you apply here, and you withdraw here, and you see what comes out here.

DUTTWEILER: An analog computer and a digital computer will do this job better than the membranes or the electrical analog.

O'CONNER: I think the crux of the thing is the model itself. Let me ask a question of both Wayne and Nick Clesciri. You each commented that you made some calculations on the model, would you go on into a little more detail relative to the nature of that. Did you assume a steady state? Annual input of so many pounds of carbon? How many steps, Wayne, did you perform in that calculation? Did you indicate that you calculated something over a 44-year period? That is a reasonable argument.

ECHELBERGER: Actually we tried various cuts at this at timed intervals to give readout of water quality, and we tried this over a quarter year period and we have allowed this to go -

O'CONNER: Over the year.

EXHELBERGER: Over a quarter of a year, yes, and we would get input and output at the end of each quarterly period and this would give us our output.

O'CONNER: What was that output?

ECHELBERGER: It was an output of phosphate concentration, suspended solids in the lake itself at that time, and organic carbon concentrations, that we indicated as chemical output.

O'CONNER: Did you go through this growth with depth to get this?

ECHELBERGER: This is what we were doing. Essentially, with charts to indicate depth. This is one cut that we did, also we did readouts on a daily basis, quarterly basis, and a yearly average of these water problem parameters. Our computer technique is a linear program.

SANDERS: One factor that we oftentimes overlook. This is the factor limiting the materials in the system. We go back to the control points limiting concentrations in substrata in your phototrophic limiting quantities of light, limiting phosphorus, limiting carbon, or whatever it happens to be. This can give us a real handle in something that we model. Too many times we develop a model which takes into consideration that these limiting factors are there. We include a factor in the model and let them go wherever they want to go. We don't put a top value on the production or on the growth rates of the organisms that we have in the system. We know all of the organisms have an optimum temperature, and groups of organisms have an optimum temperature. If we don't programme this in, if we are not cognizant of this, we are in trouble with our models.

ECHELBERGER: To answer the rest of your question: These were daily inputs, quantities of phosphorus and quantities of carbon, to maintain continuity.

O'CONNER: Is this written up any place, I would like the same process.

CLESCERI: Based on simplified models, our model was developed to mimic monthly observations for about a year's time on the chemical levels at five stations from the lake. You are almost looking at a rather slow moving stream. Five different sub-systems were developed. So it is based on the chemical levels of nitrogen and phosphorus, as well as the planktonic algae total numbers.

O'CONNER: What is the detention time?

CLESCERI: The detention time simply taking hydraulic volume is about ten years. The exact detention time is in the bailiwick of the people that are working on hydraulics. However, we feel that we are not too far off, in that the lake, chemically and biologically, is quite heterogeneous, and we do have the segment that can be part of the isolated system.

O'CONNER: Are the hydraulics people working on physical measurements?

CLESCERI: They are making current experiments in the lake, and from that they will postulate a model.

BARTSCH: I presume that in the lake you have some input basis for the productivity response. How do you go at this?

ECHELBERGER: These are some of the judgment decisions that we have made. All of the phosphorus that we have coming into the lake either as input, or was there already in the lake, is available to our algal photosynthesis.

We are also taking into account that the other nutrients that are available for this are unlimited. In our first crack at this modeling attempt this is probably not realistic, but we decided to do it this way. We are using a generation time of two days, and we have gotten an initial concentration of algal material. Looking at this from the standpoint of suspended solids, we have decided that we have about twice as many bacteria as we have algae in our system, and have made that our approach.

BARTSCH: Have you given consideration to whether or not an algal assay would be valuable in this?

ECHELBERGER: I think that an algal assay procedure would probably help us in planning some of these areas where we have difficulty.

BEETON: I agree that we should keep models as simple as possible. But also I think that if we try to model the whole lake, and find a model that will represent the lake, we have got to represent all of the limiting factors. One thing about your model that bothered me was that the consumers evidently were not represented. Even though this would make it more complex, I should think you should introduce at least some value for this, because, in lakes one of the main problems might be algal blooms, algal problems, the water intakes, and so on. It could be that it is necessary to increase the nutrients, and it may be that you change the total plankton population, and therefore this becomes a very important factor. And also I might speak to generation times. Here again, I think it might be better to generate and develop from field observations rather than those you develop from laboratory cultures, because even in southern Green Bay, for various cultures of diatoms, we have generation times of 10 days, 13 days, 10 1/2 days. The smallest generation time was 7 days.

WATTS: I find it very frustrating. I think what one has to remember is that nobody is going to give you the complete model which describes the complexities which are only individual to any particular area. The thing is, in the models, to keep working on things like this development of the model. In this model we would be the first to admit that the model is not adequate. It needs improvement, but here is what I have got, now tell me where it should be improved. Where is the place improvement is going to be best? This is the kind of comment you are making, and I think it is very valid, but it can be regarded very much as an appraisal of the model and a contribution to it, but generally it comes out sounding negative. We don't need to model the eutrophication system because there are all kinds of other systems we can model.

SCHAAKE: I really got excited by so many of these comments, and it seems to me that we should just acknowledge that it is just one complex system, and there are all sorts of things that are inter-acting with each other. We didn't really talk so much about the hydrodynamic aspects of this thing, and if some of my colleagues at M.I.T. were here, I think they would get into this act too, and discuss this matter. It seems to me that the very first diagram that we had on the board this morning, we had some statement that the real world was complex with all of the interactions, and if your modeling has put these things together in a sense of the value of things to each other, in the same sense that the design experience we talk about, if we brought things together and found that mistakes of the system in some sense that we won't worry about. The diversity of different organisms that are in the system that we don't know, we have compounded and aggregated mechanisms or functions that are taking place. The question is, what is the significance of all this. We are going to estimate parameter standards in the model, from the field measurements that we can make, or from other tests in the laboratory, we are doing the same kind of thing there. So the meaning of all this has got to be interpreted in terms of our model, and depending upon the purpose to which you put the results, we are going to either have something useful or not. The problem is, how should we do this compounding? How should we aggregate? The problem arises in every model I have ever seen.

CHEN: I want to emphasize that the hydrodynamic problem is, that. Based on our experience we feel that hydrodynamics in model building is just like mathematics to physicists. We wouldn't enter into the model construction until we had a very good handle as to how the hydrodynamics behaved in the system. So that by now we feel that we can handle the systems very well.

MOREAU: As a person who has put in a lot of time on the management side, in talking about the eutrophication process, I wonder if looking at the lake itself is really sufficient.

ECHELBERGER: I think that this is one of the objectives we are looking at with regard to the model is in the sensitivity analysis of a model. But we are dealing with a program of the various inputs and then see how the system reacts to these. By that type of analysis we can really set up a list of priorities such that if we are going to improve the quality in this body of water it doesn't matter what we do with the water itself, or with the bottom muck, the thing we really have to do is to change the land value factors in this total drainage area. Until we do this we are not going to experience any beneficial change of water quality. With the model we also might be able to say: You've got a certain time period to do this. If you can do this, say, within ten years, O.K., but if you wait any longer than ten years, no matter what you do, the lake will not recover.

VALLENTYNE: The project that we are engaged in is based on a series of about 40 lakes in a fairly isolated area in Western Ontario. These are all relatively small lakes and range from one hectare to 100 hectares in area. They vary in depth, from deep lakes to so shallow we don't attempt to classify them by depth. What we are looking at may be of some interest here. We are in a situation where we can build some interest here. We are in a situation where we can build or modify anything we like by producing it ourselves. We are trying to determine the effect on communities of the various organisms found in the water. Most of our work to date has been in developing background information on the area. What is shown is the incidence of benthic algae in the utilization of the lake.

CLESCERI: We have a compartment that considers benthic plants, the higher plants also, in production of the total biomass in the lake. We do have this factor in the model. Now, how are we going about assessing this? We aren't, at the moment, you add things on as you can. We acknowledge it as being an important potentiality.

MULTIVARIATE TECHNIQUES FOR THE CLASSIFICATION OF LAKES AND THE STUDY OF EUTROPHICATION

by

Earl E. Shannon

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MULTIVARIATE TECHNIQUES FOR THE CLASSIFICATION OF LAKES AND THE STUDY OF EUTROPHICATION

INTRODUCTION

Many criteria exist for evaluating the trophic state of lakes, and these have been presented and discussed in some detail by the previous panel. To define a lake's trophic state accurately, several parameters must be considered simultaneously. Lake trophic classification then becomes a multivariable problem and lends itself to multivariate techniques for solution.

Before continuing on the subject of trophic classification, two obvious questions should be answered. First, why is a trophic classification system needed? And secondly, what comes first, the simulation model of the eutrophication process or the classification system? In answer to the first question, a quantitative trophic classification system is necessary to:

(i) establish base lines of trophic state so that we know where we are in the dynamic process of eutrophication.

(ii) help understand better the cause-effect relationship of eutrophication and trophic state, i.e., relationship between a lake's trophic state and its environment (basin).

The second question seems to have no ready answer, and valid arguments could be presented in both directions: A good trophic classification system would require weighting of the parameters with respect to their relative importance. This, in turn, requires an in depth knowledge of how the parameters interact, i.e., a simulation model. On the other hand, a useful model would probably be restricted to a narrow range of lake types, e.g., possibly one model is needed for each trophic state. Presumably even a crude classification system could serve as a framework for sampling and experimental design which would supply information for better models, which in turn would feed back information to upgrade the classification system. This situation is depicted diagramatically in Figure I.

Many of the trophic indicator (classification)schemes are univariate i.e. consider one trophic index. Examples of such trophic indicator schemes proposed and used by researchers include:

(i) nutrient levels at spring turnover, (Sawyer, 1947)
(ii) algal indicators, (Brook, 1965) and (Rawson, 1956)
(iii) lake moan depth, (Rawson, 1952)
(iv) changes in abundance and species of fish population, (Rawson, 1952) and (Beeton, 1960)
(v) zooplankton indicators, (Jarnefelt, 1952)
(vi) bottom fauna changes, (Beeton, 1960) and (Verduin, 1964)
(vii) average Secchi disc reading, (Hasler, 1947)
(viii) dissolved solids and conductivity measurements,(Minder, 1918) and (Beeton, 1960)
(ix) productivity measurements by various methods, (Ryther, 1956).

Numerous other schemes could also be enumerated, but those listed represent the more common systems. The chief advantages of a univariate approach to trophic classification is its simplicity and ability to indicate trophic state in a qualitative sense. The disadvantages of a univariate scheme are several. The indicies are for the most part qualitative. They are narrow in scope, (i.e., apply to special situations) and above all

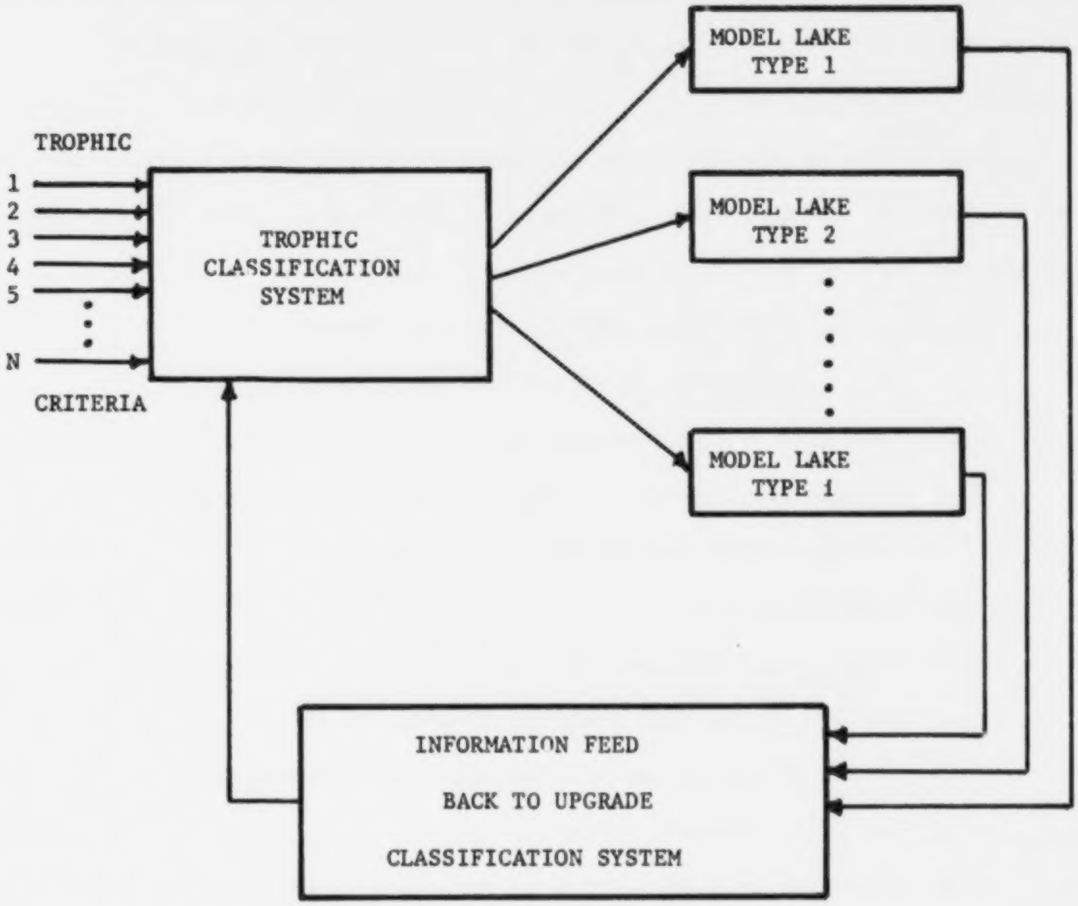
TROPHIC
1
2
3
4
5
N
CRITERIA
TROPHIC CLASSIFICATION SYSTEM
MODEL LAKE TYPE 1
MODEL LAKE TYPE 2
MODEL LAKE TYPE i
INFORMATION FEED BACK TO UPGRADE CLASSIFICATION SYSTEM

FIGURE I

TROPHIC CLASSIFICATION SYSTEM AND SIMULATION MODEL INTERACTION consider only a part of the complex of variables that describe and determine trophic state.

It should be noted that few investigators use a single trophic index in classifying a lake, e.g., Beeton, (1960) considered many trophic characteristics of the Great Lakes in assessing their trophic states. However, the individual indicies tend to give conflicting results, thereby making it difficult to classify a lake using a qualitative scheme.

Recent work by Huff, Watts et al (1969) at the University of Wisconsin illustrates the complexity of just one part of the eutrophication process, i.e., the transport and transformation of water and nitrogen variables in a lake basin. Water variables in the lake basin may be transported, transformed, or stored. Table I provides a listing of the different forms of water variables which can be transported, transformed, or stored throughout the lake basin system. Reading the table horizontally shows that variable No. 13 produces water in the form of variables 12, 13, 15, 16, 17, 19, and 20. The directional arrows indicate feed forward or feed back. The nitrogen variables are summarized in Table II with their associated carrier materials (water, biomass, and atmosphere). Reading across the chart provides information on which forms of nitrogen can exist in a carrier material variable; for example, surface water (13) may contain nitrogen in the form of dissolved N_2, NH_3, NH_4^+, NO_2^-, NO_3^-, and NR. Reading down the chart provides information on which variables each form of nitrogen has; for example, NH_4^+, can only exist in water variables 13 and 24, not in any biomass variables.

This brief illustration of the intricate nature of one part of the eutrophication process should indicate the virtual impossibility of simulating the process as a whole. However, simple sub-component models in conjunction with multivariable trophic classification schemes may provide valuable insight into the process.

It must be recognized that no trophic classification system will be universal but perhaps the most logical approach to lake classification has been suggested by Zafar (1959). His nomenclature system consists of three components: the first indicates the physico-chemical nature of the water; the second, the climatic zone and the type of lake basin; while the third, gives an idea of the dominant class of organisms and the productivity in general. An example of a lake named by Zafar's system is " γ - eutrophic - β - tropico-absolutae Bacillariae", where γ - eutrophic refers to the chemical nature of the water, β to the shape of the lake basin, tropico-absolutae to the geographic location and elevation and Bacillariae to the dominant class of phytoplankton present in the lake. Zafar's system is qualitative in that it is concerned only with naming lakes, but with the addition of other important trophic criteria and the use of multivariate techniques it is possible the basic nomenclature system could be broadened into a quantitative trophic classification system.

The rest of this paper describes some multivariate techniques that may hold potential for solving the trophic classification problem and studying the variable interactions in the eutrophication process in general.

MULTIVARIATE METHODS

Multivariate statistical analysis is concerned with data collected on several dimensions of the same individual or object. The basic difference between many multivariate methods and their univariate prototypes is that the multivariate methods work with vectors and matrices rather than scalar quantities. The use of multivariate techniques has expanded with the development of the digital computer, since they lend themselves to the large storage and rapid execution features of the computer.

This section contains a description of several multivariate techniques accompanied by illustrative examples. No attempt is made to go into the theory of each method. However, a brief description of the basic principles involved is given. Many texts are available that provide in depth treatments of the theory and application of multivariate methods i.e. Anderson (1958), Morrison, (1967), Cooley and Lohnes (1962), Roy (1957), Rao (1952), and Kendall (1961).

The data used in all the examples are drawn from a eutrophication research project being conducted in the Department of Environmental Engineering at the University of Florida. In this project, fifty-five north Florida lakes are being extensively studied. These lakes represent a cross section of trophic states including oligotrophic, mesotrophic, eutrophic, hypereutrophic, senescent, and dystrophic forms. Surface areas and depths of the lakes range from .9 hectares to 12,000 hectares and 1.5 meters to 25 meters respectively. Biological, chemical, and physical parameters have been determined periodically on these lakes for two years. Table III summarizes these parameters in addition to showing the morphometric characteristics that have been evaluated. One of the objectives of this study is to supply data for a multivariable trophic classification system and as shown in Table III, virtually all important trophic criteria have been evaluated. All the raw data used in any of the examples are given in Appendix A.

Cluster Analysis

Cluster analysis is concerned with the problem of classifying objects into groups based on a collection of data describing the objects, when the number of groups that best fit these data is not predetermined. In essence, the method attempts to pick out logical groupings in the N-dimensional hyperspace described by the N data attributes of the object. The procedure is not a classical multivariate technique but it holds application in the problem of lake classification, since we can determine quantitatively, groups of lakes that are of similar trophic status. These groups may then be interpreted and accordingly named on a trophic scale (or any other meaningful nomenclature system) ranging from oligotrophic to hypereutrophic assuming that all forms were represented in the original sample group. It is possible that the groups delineated could be used in the calculation of discriminant functions (discussed in a later section). These discriminant functions could then be used to classify lakes outside the sample group by considering their pertinent trophic characteristics.

The three basic elements of a cluster analysis study are:

(i) the between object distances
(ii) the clustering criterion
(iii) the computational procedure.

Most popular distances employed are the correlation coefficient between the objects and some simple function of the Euclidean distance. Clustering criteria (measure of the goodness of any given allocation) usually include a measure of within group similarity. In some cases, good within group similarity implies good between group dissimilarity. Computational procedures are usually of the heuristic type in the interest of solving large problems with an economy of computer time. Heuristic methods are intuitively designed procedures developed to give good solutions to a problem without guarantee of ever reaching an optimum.

Many clustering schemes have been devised by using different distance measures, clustering criteria and computational procedure. Ball and Hall (1965), Fortier and Solomon (1966), Friedman and Rubin (1967), and King (1967) have all proposed and tested basically different schemes. The clustering method used in the example presented here was developed by Padron (1969) at the University of Florida where a computational algorithm was used as a guide in devising a rapid devised heuristic computational procedure. The program was written in Fortran IV and was used on the IBM 360/65 computer. The example presented (Table IV) illustrates the use of this cluster analysis program for classifying the fifty-five lake sample into groups considering nine trophic characteristics of the lakes.

In Table IV, the trophic characteristics used in the grouping are listed. The optimum number of groups indicated by the analysis was seven. The lake numbers falling in each group are listed together with a qualitative trophic classification of the lakes in parenthesis. This qualitative classification of the study group lakes was made by Brezonik (1969) before the cluster analysis was performed. From the cluster analysis and the qualitative classification, the groups might be tentatively named as:

Group I - Oligo-Mesotrophic
Group II - Mesotrophic
Group III - Hypereutrophic
Group IV - Eutrophic

Group V - Residual Group
Group VI - Dystrophic
Group VII - Oligotrophic

The existence of this residual group points out a disadvantage of the clustering procedure, namely, residual groups may be formed containing members that are alike only in being different from the other groups. Perhaps if additional trophic characteristics were included in the analysis; for example, sediment analysis, phytoplankton counts, and species diversities, the groupings may be somewhat different and easier to interpret. Inclusion of color in the characteristics might have helped to separate lakes into trophic-color groups such as oliogrophic-high color and oligotrophic-low color. However, it should be noted that most of the oligotrophic colored lakes have been placed in one group even though color has not been included as a characteristic.

Cluster analysis may also be used to group trophic characteristics, i.e., indicate characteristics which tend to measure similar properties of the lake. Goldman et al (1968) have carried out a cluster analysis on the variables influencing primary production and found meaningful groupings such as solar radiation and water temperature. In the present study, a cluster analysis on the nine trophic characteristics (Table IV) indicated that the optimum number of groups of characteristics is seven with primary production and chlorophyll being placed into one group and total solids and conductivity into another group. The six other characteristics remained in groups of one.

Canonical Correlation Analysis

Canonical correlation analysis is a method of describing the dependence between two sets of variates. The technique is due to Hotelling (1939).

Observations are considered in vector form where the vector X consists of n individual variates measured on the same object at the same time. For example, an observation vector might consist of various trophic characteristics measured on a lake on a particular day. This vector is assumed to be from a population which is distributed multivariate normal with mean vector μ and covariance matrix Σ . Suppose certain variates within the vector X possess some common characteristics. For example, consider a four component trophic characteristic vector consisting of biological activity indicators primary production and chlorophyll a and the nutrient factors total phosphate and total nitrogen. The vector X may then be subdivided into two distinct smaller vectors, $\underline{X}_1$ and $\underline{X}_2$ with p=2 and q=2 components, respectively, where $\underline{X}_1$ is the biological activity vector and $\underline{X}_2$ the nutrient vector. (Note that vector quantities are underscored). If N ($N \geq p+q+1$) independent observation vectors are drawn from the population of lakes and the sample covariance matrix S is calculated it too may be partitioned as was X.

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

If testing demonstrates that there is in fact dependence between the two sets of variates $\underline{X}_1$ and $\underline{X}_2$ (procedure not described here) the method of canonical correlation may be applied. In canonical correlation, one is interested in the relationship between the two pairs of variates $\underline{X}_1$ and $\underline{X}_2$. The following question may be proposed.

What are the linear compounds

$$\begin{array}{ll} W_1 = \underline{a}_1' \underline{X}_1 & Z_1 = \underline{b}_1' \underline{X}_2 \\ \vdots & \vdots \\ W_s = \underline{a}_s' \underline{X}_1 & Z_s = \underline{b}_s' \underline{X}_2 \end{array}$$

with the property that the sample correlation of W_1 and Z_1 is greatest, the sample correlation of W_2 and Z_2 is greatest among all linear compounds uncorrelated with W_1 and Z_1 and so on for all $s = \min(p,q)$ possible pairs? It can be shown that the coefficients of the i^{th} pair are given by the homogeneous equations (Morrison, (1967)

$$(S_{12} S_{22}^{-1} S_{21} - C_i S_{11}) \underline{a}_i = 0 \tag{1}$$

$$(S_{21} S_{11}^{-1} S_{12} - C_i S_{22}) \underline{b}_i = 0 \tag{2}$$

where C_i is the i^{th} largest root of the determinantal equations

$$|S_{12} S_{22}^{-1} S_{21} - \lambda S_{11}| = 0 \quad \text{or}$$

$$|S_{21} S_{11}^{-1} S_{12} - \lambda S_{22}| = 0 \tag{3}$$

By computing the sample covariance matrix, S, or its counterpart the correlation matrix, R and partitioning as previously indicated it is possible to find highly correlated pairs of new variates that are combinations of the original variates. This has been done in an example (Table V) for a biological activity vector $\underline{X}_1$ and a nutrient factors vector $\underline{X}_2$ using data gathered on the fifty-five lakes during December of 1968. The raw data is listed in Appendix A. The canonical correlation program used with the IBM 360/65 computer was the BMDO6M program from the Biomedical Computer Programs Library (1968).

Results obtained in the example indicate that primary production and chlorophyll have nearly equal weights in determining the biological activity variable, whereas in its correlative variable of nutrient factors, total organic nitrogen has about twenty times more weight than does total phosphate. Based on this, it appears that the major link between the two highly correlated canonical variables is between primary production plus chlorophyll a and total organic nitrogen. Direct interpretation of the results of a canonical analysis as being indicative of causal mechanisms can be risky. However causal mechanisms may be suggested and in many cases are verified by deterministic findings.

One of the problems confronted in studying interparameter relationships in the eutrophication process is defining dependent variables. For example, in using multiple regression analysis one dependent variable (e.g. primary production)is designated and is considered as a function of several independent variables (e.g. nutrient and physical factors). The independent variables are usually not strictly orthogonal, i.e., nutrient components are at least to some extent related to each other. This limits the statistical validity of multiple regression analysis as a predictive tool. However, canonical analysis is useful in combining variables that are highly correlated so that in essence more than one variate can be linearly combined into a dependent variable.

Canonical correlations between several sets of variates has been studied by Horst (1961), Anderson (1958) and Roy (1957). Perhaps this approach holds promise for investigating correlative relationships between sets of biological, physical, and chemical variables in a lacustrine system. Another use of canonical analysis may be in establishing relationships between lacustrine trophic state and basin parameters influencing the trophic state.

Factor Analysis

Factor analysis aims to explain observed relations among numerous variables in terms of simpler relations. More formally factor analysis may be defined as the isolation and development of hypothetical constructs out of observed phenomena. Cattell (1965) states that the most valuable application of factor analysis appears to lie in the fields of biological and behavioral sciences where a great array of phenomena are simultaneously determined and where the conceptual independent variables are not easily located and agreed upon. Factor analysis was introduced by Spearman (1904) and in the following years the development and application of the analysis has been confined primarily to the field of psychology.

Opinions on the usefulness of factor analysis are somewhat mixed since the method is somewhat underdeveloped technically. Ehrenberg (1962), for example, takes an antagonistic viewpoint and says that factor analysis is often thought to be something it isn't. Matalas (1967) in commenting on the usefulness of factor analysis in studies of hydrologic data states that at best factor analysis may be useful as an exploratory tool to see how the variables group themselves. Cattell (1965) takes a more optimistic viewpoint and states that factor analysis makes available a generalized method for making invisible influences visible at least to a first approximation of their form. Arguments concerning the pros and cons of factor analysis have not prevented its widespread use in practically all fields of science. Application in the field of lake eutrophication and classification should provide valuable insight. No attempt will be made to go into the theory of factor analysis and thereby point out some of its technical faults. The important point to realize is that the method does have weaknesses and because of these, its capabilities are limited, A brief review of the methodology of factor analysis follows.

The basic factor model may be expressed as

$$X = F. A' + U.D \tag{4}$$

where X = a matrix of standardized observations variables with dimensions (Nxp)

F = a matrix of uncorrelated factor values (Nxr)

A = a matrix of factor loadings (pxr)

U = a matrix of standardized uncorrelated "unique variables" also uncorrelated with the F variables (Nxp)

D = a diagonal matrix with diagonal elements d_i (pxp)

N = number of observations

p = number of variables

r = number of factors

' = denotes the transpose of a matrix

Factor analysis takes the explained variance in the correlation matrix, R_x and redistributes it among a set of factors that reveal underlying linear combinations of the original variables. The splitting up of the variance may be represented as

$$R_x = AA' + D^2 \tag{5}$$

Considering the j^{th} diagonal element of R_x gives the expression:

$$\sum_{k=1}^{r} a_{jk}^2 + d_j^{\,2} = 1 \tag{6}$$

The first term of (6) represents what the j^{th} variable has in common with the other $p-1$ variables. This is the proportion of the variance that is explained by the factors and is called the "communality". The second term of (6) represents the proportion of the variance not explained by factors and is called the "uniqueness" property.

The first step in factor analysis is to find the principal components (eigenvectors) of the correlation matrix R_x. The first principal component is defined as that combination of the x_j's that explains the greatest possible amount of the variance in the correlation matrix. The second principal component is chosen as an independent combination of the x_j's which then explains as much as possible of the remaining variance and so forth. By this method as many principal components can be extracted from the correlation matrix as there are variables. The principal components are orthogonal and if considered as a new set of variables, they represent a set of independent variables. Although no section of this paper is devoted to principal component analysis, it should be mentioned that it also has considerable potential as a tool for use in the lake classification problem.

Factor analysis, from the principal components, derives a set of factors that are considered to of "simple structure". "Simple structure" means that each factor has the greatest interpretability or simplicity in that its coefficients (a_{ij}) tend either to unity or zero. Thus, a given factor either does or does not include a particular variable. This may be accomplished by different methods but in this study a Kaiser (1958) varimax rotation was used.

An example of factor analysis in interpreting trophic characterics is given in Table VI. The program used with the IBM 360/65 computer was BMDO3M from the Biomedical Computer Programs Library. The data analyzed were measures of nine trophic characteristics made on the fifty-five lake sample group referred to before. (See Appendix A).

In the example, four factors have been arbitrarily selected since they explain most of the variance (91%) and can be interpreted readily. However, there are other methods available which one can use in determining how many factors should be employed (See Cattell 1965). What the factors represent depends somewhat upon the interest of the investigator. In the example Factor I has been interpreted as a biological activity factor since very high loadings are placed on primary production and chlorophyll a variables. Factor II is interpreted as a limiting nutrient factor since high loading is placed on the total phosphate variable. Factor III is interpreted as a factor indicating the physical nature of the lake water and Factor IV interpreted as a general nutrient factor since the nutrient index has a high loading.

Interpreting the factor analysis results further indicates that the measures of productivity, e.g., chlorophyll a and primary production appear to be the most important trophic characteristics (of the nine considered) in that they explain a large percentage of the total variance. Other important characteristics are total phosphate, Secchi disc, and cation index as indicated by the II, III and IV factors. In a classification system incorporating several trophic characteristics one would expect to weight productivity heaviest followed by a lighter weighting of the other trophic characteristics. The major contribution of factor analysis in classification systems seems to be in suggesting weightings for the trophic characteristics considered by the classification system. Principal component analysis may be useful in reducing the dimensions of the trophic classification system in that several variables may be combined into a smaller number of interpretable principal components.

Discriminant Analysis

In classification the question arises: Given an individual (lake) with certain measurements (trophic characteristics), from which population (trophic group) did it come? One method in multivariate analysis, useful for classification purposes is discriminant function analysis. This method differs from cluster analysis (discussed earlier) in that information needs to be known about the population parameters (mean vector and covariance matrix) from which the groups arise. At least estimates of the population parameters must be available. In order to get estimates of the population parameters usually a qualitative knowledge is required of what individuals lie in each group. Anderson (1958) has presented an excellent chapter on the classification of observations in his book. In general, the problem can be described as follows:

> Let π_1, π_2, ...π_m be m populations with density functions $p_1(\underline{x})$, $p_2(\underline{x})$ ....$p_m(\underline{x})$. (Where $\underline{x}$ is an observation vector)

We wish to divide the space of observations into m mutually exclusive and exhaustive regions R_1, R_2, ...R_m. If an observation falls into R_1

we shall say that it comes from π_i. Let the distribution of π_i be $N(\mu^{(i)}, \Sigma)$ where $\mu^{(i)}$ is the mean of π_i and Σ the common covariance matrix for all i populations. Assume that the costs of mis-classification are equal and the a priori probabilities, q_i, are known, then the region R_i is defined by those $\underline{x}$ satisfying

$$R_i: u_{ik}(x) \geq \log \frac{q_k}{q_i}, \quad k = 1, \ldots\ldots\ldots\ldots, m; \; k \neq i \tag{7}$$

where

$$u_{ik} = \frac{\log p_i(\underline{x})}{\log p_k(\underline{x})} = [\underline{x} - 1/2 \, (\underline{\mu}^{(i)} + \underline{\mu}^{(k)}]' \; \Sigma^{-1}(\underline{\mu}^{(i)} + \underline{\mu}^{(k)}) \tag{8}$$

$u_{jk}(x)$ is the discriminant function related to the j^{th} and the k^{th} populations when the population means $\mu^{(i)}$ and $\mu^{(k)}$ and population covariance matrix Σ are known. q_k and q_i are the a priori probabilities of $\underline{x}$ being from population k or i respectively. Usually $\mu^{(i)}$ and Σ are not known and $\underline{x}$ the sample mean vector and S the sample covariance matrix are used as their estimates. The discriminant function then becomes

$$v_{ik}(\underline{x}) = [\underline{x} - 1/2 \, (\underline{\bar{x}}^{(i)} + \underline{\bar{x}}^{(k)})]' \; S^{-1}(\underline{\bar{x}}^{(1)} + \underline{\bar{x}}^{k}) \tag{9}$$

where $v_{jk} \to u_{jk}$ when the number of observations $N_i \to \infty$. Hence for sufficiently large samples one may consider v_{jk} as a good estimator of u_{jk}. Discriminant functions are also defined for the situation where a priori probabilities are not defined. Costs of misclassification may also be introduced into the discriminant function.

As an example of the usefulness of discriminant analysis, discriminant functions of the form of equation (9) were obtained for four groups of lakes using the same nine trophic characteristics as in the previous example on factor analysis. The lake trophic groups were determined partly on the basis of cluster analysis results for four groups and partly on an intuitive knowledge of their trophic states. The four groups were arbitrarily named Oligotrophic I, Oligotrophic II, Mesotrophic, and Eutrophic where Oligotrophic II denotes a more advanced trophic state than Oligotrophic I but less than Mesotrophic. A stepwise discriminant analysis was performed (bringing trophic characteristics into the discriminant function one at a time) using the IBM 360/65 computer and the Biomedical Computer Program BMD07M. The resulting discriminant functions are shown in Table VII. Two lakes from outside the 55 lake sample group (Ten-a and Twelve-a) were tested in the discriminant functions to see what trophic populations they came from. One lake (Ten-a) was classified as Oligotrophic II the other lake (Twelve-a) was classified as Oligotrophic I.

Discriminant analysis, as has been shown, provides a quantitative method of telling from what trophic group a particular lake comes provided the sample of lake groups used to establish the discriminant functions is truly representative of the trophic populations. Of course, this is the case with any statistical procedure, i.e., the validity of test depends on how well the sample represents the population. Discriminant analysis, although it requires some previous knowledge of groupings, seems to hold more potential for classifying lakes than does cluster analysis. Cluster analysis results in logical groupings but these groups in turn require interpretation which, in essence involves some prior knowledge of a lake's trophic status. Discriminant analysis provides a linear function of the trophic criteria used in classification. From this function, especially when stepwise discriminant analysis is used, it is possible to pick out the more important variables in differentiating between groups. Probabilities of misclassification can be calculated in discriminant analysis to give an idea of how good the method is for a particular classification problem, whereas there is little way of knowing the worth of a particular cluster analysis. Costs of misclassification can be incorporated in discriminant analysis thereby offering some protection against the undesirability of a particular misclassification.

Other Multivariate Methods

In the previous parts of this paper some of the more common multivariate techniques thought to be useful in lake classification and analysis of the eutrophication process have been discussed. However, there are other useful methods.

The multivariate analysis of variance could prove to be useful in determining significance differences among a particular set of trophic groups as well as those variables contributing most to the differences. Profile analysis may indicate whether, statistically speaking, trophic characteristics behave the same in all trophic groups.

The use of multiple regression techniques must not be overlooked. Their major use appears to be as a predictive tool when a dependent variable can be defined. An example is predicting primary production levels in a lake knowing "independent" variables such as nutrient levels, solar radiation, water temperature, turbidity, etc. Its potential in such areas has been shown by several investigators including Goldman et al (1968) and Brezonik et al (1969).

Other multivariate methods that have not been mentioned may also prove to be of use in analysis of the eutrophication process. No attempt has been made in this paper to exhaustively discuss all the available techniques. However, most of the major ones have been presented.

SUMMARY

In this paper lake trophic classification has been discussed as a multi-variable problem. Some potentially useful multivariate techniques for solution of this problem and that associated with lake eutrophication have been presented. Major points in the paper are summarized as follows:

(i) Lake trophic classification and the study of eutrophication are multivariable problems and lend themselves to multivariate techniques for solution.

(ii) Simulation models of the eutrophication process and trophic classification systems are complementary in that better trophic classification systems result in better models and vice versa.

(iii) Cluster analysis offers a method for forming logical trophic groups of lakes when little is known about the groups beforehand.

(iv) Canonical analysis is a useful multivariate procedure with potential for studying in a statistical sense parameter interrelationships in the lacustrine system and the relationship between a lake and its environment (basin).

(v) Factor analysis and principal component analysis are possible methods available for weighting the trophic criteria used in a classification system. In some cases they may prove useful for reducing the dimensions of a classification system by combining several criteria into one factor or component.

(vi) Discriminant analysis by providing a set of discriminant functions offers a means of determining the trophic state of a particular lake.

(vii) Interpretation of statistical results in terms of cause-effect relationships must be done with caution, especially in the eutrophication process where a multitude of variable interrelationships are involved. However, multivariate techniques do provide the researcher with valuable tools for making inferences and predictions concerning the lacustrine system.

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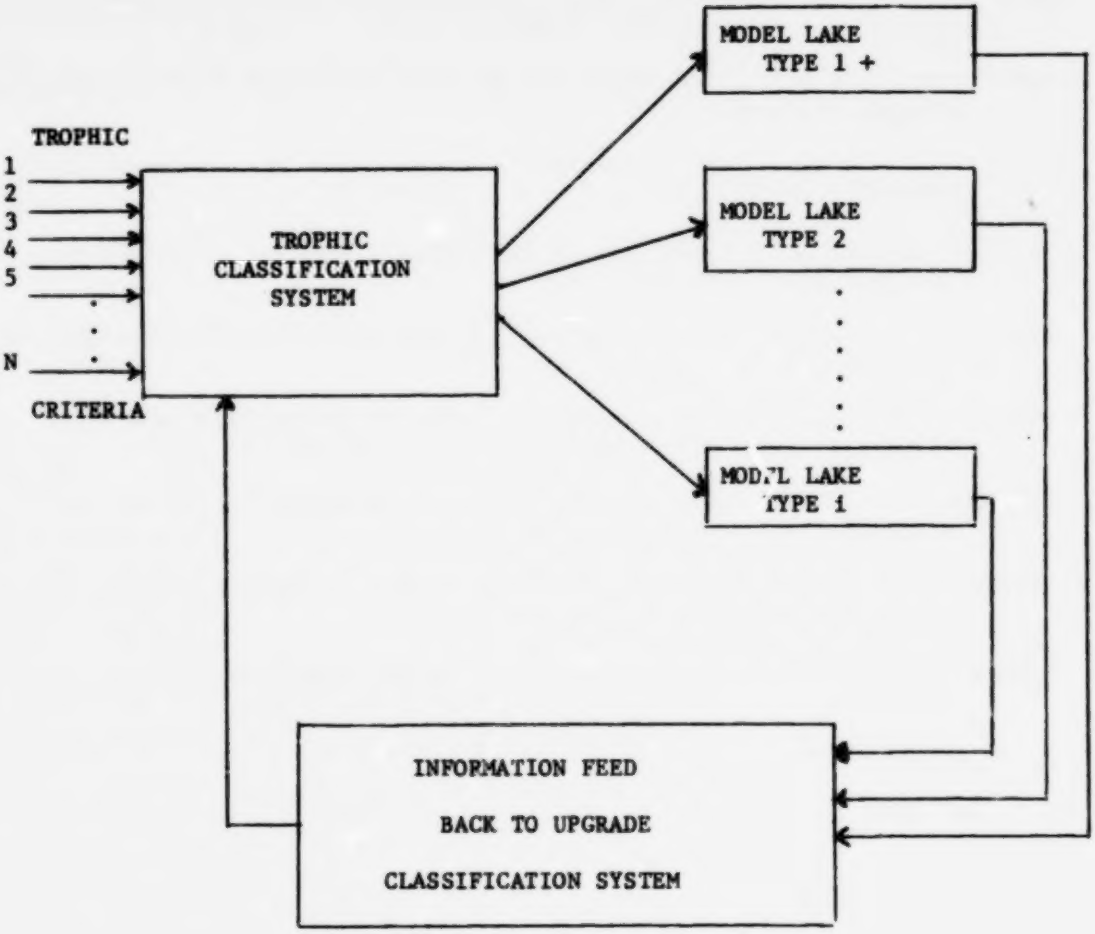
TROPHIC
1
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N
CRITERIA
TROPHIC CLASSIFICATION SYSTEM
MODEL LAKE TYPE 1 +
MODEL LAKE TYPE 2
MODEL LAKE TYPE i
INFORMATION FEED BACK TO UPGRADE CLASSIFICATION SYSTEM

FIGURE I

TROPHIC CLASSIFICATION SYSTEM AND SIMULATION MODEL INTERACTION

WATER VARIABLES		WATER TRANSFORMATIONS 11	12	13	14	15	16	17	18	19	20	21	22	23	24
PRECIPITATION	11	0	→	→				→		→	→				
VAPOR	12		0	→											
SURFACE	13		←	0		→	→	→		→	→				
TERRESTIAL VEGETATION	14		←		0	→									
TERRESTIAL PRIMARY CONSUMERS	15		←	←		0	→								
TERRESTIAL SECONDARY CONSUMERS	16		←	←			0								
ROOT INTERSTITIAL	17				←			0	→	→	→				
INTERMEDIATE INTERSTITIAL	18							←	0	→	→	→			
STREAM	19		←					←	←	0	→	→	→	→	→
EPILIMNION LAKE	20		←					←	←	←	0	→	→	→	→
HYPOLIMNION LAKE	21							←	←			0	→	→	→
AQUATIC VEGETATION	22												0	→	
AQUATIC PRIMARY CONSUMERS	23									←	←	←		0	→
AQUATIC SECONDARY CONSUMERS	24									←	←	←			0

TABLE I

WATER VARIABLE TRANSFORMATIONS
(Feed Forward and Feedback)
[From Huff, *et al* (1969)]

TABLE II

[From Huff, et al (1969)]

NITROGEN VARIABLES

No.	NAME	NO	N_2	NH_3	NH_4^+	NO_2^-	NO_3^-	NR
1	Gas	1	1	1				
2	Particles in air					2	2	2
11	Precipitation	11	11	11			11	11
12	Vapor							
13	Surface water		13	13	13	13	13	13
14	Terrestrial vegetation water		14	14	14		14	14
15	Terrestrial primary consumers water		15		15		15	15
16	Terrestrial secondary consumers water		16		16		16	16
17	Root interstitial water		17	17	17	17	17	17
18	Intermediate interstitial water		18	18	18	18	18	18
19	Stream water		19	19	19	19	19	19
20	Epilimnion lake water		20	20	20	20	20	20
21	Hypolimnion lake water		21	21	21	21	21	21
22	Aquatic vegetation water		22	22	22	22	22	22
23	Aquatic primary consumers water		23	23	23		23	23
24	Aquatic secondary consumers water		24	24	24		24	24
	NITROGEN IN BIOMASS							
41	Terrestrial vegetation						41	41
42	Terrestrial vegetation standing dead						42	42
43	Terrestrial vegetation detritus						43	43
44	Terrestrial primary consumers							44
45	Terrestrial secondary consumers							45
46	Terrestrial animal							
47	detritus			47			47	47
48	Terristrial fungi			48			48	48
49	Terrestrial bacteria			49		49	49	49
50	Terrestrial microfauna						50	50
51	Insoluble soil organic						51	51
52	Inorganic particles					52	52	

Table II continued

No.	NAME	NO	N_2	NH_3	NH_4^+	NO_2^-	NO_3^-	NR
61	Aquatic vegetation						61	61
62	Aquatic vegatation standing dead						62	62
63	Aquatic vegetation detritus						63	63
64	Aquatic primary consumers							64
65	Aquatic secondary consumers							65
66	Aquatic animal detritus						66	66
67	Aquatic algae			67			67	67
68	Aquatic fungi			68			68	68
69	Aquatic bacteria			69		69	69	69
70	Aquatic scavengers						70	70
71	Aquatic insoluble organic						71	71

TABLE III

PARAMETERS MEASURED IN A STUDY OF FIFTY-FIVE NORTH FLORIDA LAKES

PHYSICAL

Temperature Profile
Population characteristics of drainage basin
Secchi Disc Transparency

MORPHOMETRIC

Lake Mean Depth
Lake Area
Watershed Area
Soil Types in Watershed
Land Use in Watershed
Shoreline Development

CHEMICAL

A. WATER

Acidity
Alkalinity
Calcium
Chloride
COD
Magnesium
Manganese
Organic Nitrogen
Ammonia
Nitrite
Total Phosphate
Potassium
Silica
Sodium
Suspended Solids

Table III continued

CHEMICAL

A.Contd.

- Color
- Conductance
- Fluoride
- Iron
- Nitrate
- Oxygen, Dissolved
- pH
- Orthophosphate
- Total Solids
- Sulfate
- Turbidity

B. Sediments

- Ammonia
- Organic Nitrogen
- Total Phosphate
- Sediment Type (Visual Classification)
- Plant Pigment Derivatives
- Percent Volatile Solids
- Iron
- Manganese
- C, H, N Elemental Analysis

BIOLOGICAL

- Algal Identification and Counts
- Chlorophyll a, b, and c
- Total Carotenoids
- Primary Production
- Species Diversities Indicies of Algae
- Benthic Organisms

TABLE IV

CLUSTER ANALYSIS ON FIFTY-FIVE NORTH FLORIDA LAKES CONSIDERING NINE TROPHIC CHARACTERISTICS

TROPHIC CHARACTERISTICS

(i) Secchi Disc (ft)
(ii) Total Organic Nitrogen (mg/l)
(iii) Total Phosphate (mg/l)
(iv) Specific Conductance (μmho/cm)
(vi) Chemical Oxygen Demand (mg/l)
(vii) Primary Productivity (mgC/hr/m^3)
(viii) Chlorophyll a (mg/m^3)
(ix) Cation Index(1)

Optinum Number of Groups Indicated by Program = 7

Table IV continued

Group I

1	(OC)(2)	11	(MC)
2	(OC)	12	(MC)
3	(OC)	13	(-)
4	(MC)	46	(OC)
6	(MC)	48	(MC)
9	(MC)	51	(MC)
10	(MC)		

TOTAL 13 Lakes

Group II

21	(M)
24	(EC)
28	(E-S)
31	(EC)
32	(E)
33	(MC)
39	(M-E)

TOTAL 7 Lakes

Group III

17	(HE)
20	(HE)
23	(HE)
35	(HE)
37	(E)
38	(E)
34	(HE)

TOTAL 7 Lakes

Group IV

8	(EC)
14	(E)
15	(E)
16	(D-S)
19	(EC)
30	(D-OC)

TOTAL 6 Lakes

Group V

5	(M)
22	(S)
25	(M)
36	(E)

TOTAL 4 Lakes

Group VI

7	(M)
18	(D-OC)
26	(D-MC)
27	(M)

TOTAL 4 Lakes

Group VII

29	(D-OC)	47	(0)
40	(0)	48	(0)
41	(0)	50	(0)
42	(0)	52	(0)
43	(0)	53	(0)
44	(0)	54	(0)
45	(0)	55	(0)

TOTAL 14 Lakes

(1) $\frac{Na + K}{Ca + Mg}$ Pearsall's (1923) Basic ratio

(2) Symbols are as follows:

0 = Oligotrophic
M = Mesotrophic
E = Eutrophic
HE = Hypereutrophic
D = Dystrophic
S = Senescent
C = High Organic Color in Lake Water

TABLE V

CANONICAL CORRELATION ANALYSIS RUN ON DATE COLLECTED IN DEC. 1968 ON FIFTY-FIVE NORTH FLORIDA LAKES

N = # of Lakes = 55
P = # of Variables in X_1
Biological Activity Vector = 2
Q = # of Variables in X_2
Nutrient Factors Vector = 2

Biological Activity Vector Consists of:	Primary Production (PP) and Chlorophyll a (CHA)
Nutrient Factors Vector Consists of:	Total Nitrogen (TON) and Total Phosphate (TP4)

Correlation Matrix R:

TON	TP4	PP	CHA	
1.0000	.5715	.7914	.8081	TON
.5715	1.0000	.4536	.4986	TP4
.7914	.4536	1.0000	.8666	PP
.8081	.4986	.8666	1.0000	CHA

First Canonical Correlation = .8290

Biological Activity Variable: .44 PP + .60 CHA
Nutrient Factors Variable: .98 TON + .04 TP4

TABLE VI

FACTOR ANALYSIS ON NINE TROPHIC CHARACTERISTICS OF 55 NORTH FLORIDA LAKES

CORRELATION MATRIX R_X:

	Secchi Disc	Conductance	Total Organic Nitrogen	Total Phosphate	Nutrient Factor	Chemical Oxygen Demand	Total Solids	Primary Production	Chlorophyll a
1	1.0000	-.33480	-.54699	-.42724	.35368	-.68804	-.50666	-.31547	-.46476
2		1.0000	.58690	.74200	-.62884	.45356	.80143	.55077	.58722
3			1.0000	.57147	-.54499	.85433	.56194	.79135	.80805
4				1.0000	-.55167	.50439	.81838	.45366	.49861
5					1.0000	-.50186	-.63150	-.37538	-.47530
6						1.0000	.49925	.70171	.70608
7							1.0000	.42166	.48643
8								1.0000	.86662
9									1.0000

ROTATED FACTOR MATRIX (FOUR FACTORS) AND INTERPRETATION

I.	-.19101	.33140	.68820	.24261	-.20837	.58897	.18734	.93483	.88343
II.	-.18116	.48193	.24675	.90474	-.26438	.18813	.64528	.16982	.19019
III.	-.94768	.09253	.30835	.18190	-.14645	.53274	.27587	.06846	.24963
IV.	-.11226	.30137	.23729	.21206	-.90472	.23608	.30947	.08596	.20140

Table VI continued

INTERPRETATION OF FACTORS

Factor I - Biological Activity Factor

Factor II - Limiting Nutrient Factor

Factor III - Physical Characteristic of Water Factor

Factor IV - General Nutrient Factor

Four Factors Explain 91% of the Total Varience

TABLE VII

DISCRIMINANT FUNCTIONS FOR FOUR TROPHIC GROUPINGS OF LAKES - OLIGOTROPHIC I, OLIGOTROPHIC II, MESITROPHIC AND EUTROPHIC

TROPHIC CHARACTERISTICS INCLUDED IN THE DISCRIMINANT FUNCTIONS:

(i) Secchi Disc(SD) (Ft)
(ii) Conductivity (COND)(μmho/cm)
(iii) Total Organic Nitrogen(TON)(mg/L)
(iv) Total Phosphate (TP4) (mg/L)
(v) Cation Index (CI)
(vi) Chemical Oxygen Demand(COD)(mg/L)
(vii) Total Solids (TS)(mg/L)
(viii) Primary Production (PP)(mgC/hr/m^3)
(ix) Chlorophyll a (CHA)(mg/m^3)

DISCRIMINANT FUNCTIONS:

$U_{\text{Oligo I, Oligo II}}$ = -9.359 + 2.280 (SD) -.045 (COND) + 1.014 (TON)
-10.257 (TP4) - 5.674 (CI) -.090 (COD)
+ .011 (TS) + .031 (PP) -.163 (CHA)

$U_{\text{Oligo II, Meso}}$ = 9.186 - .176 (SD) -.051 (COND) + 2.161 (TON)
-18.247 (TP4) + .910 (CI) -.158 (COD)
+ .035 (TS) + .048 (PP) -.332 (CHA)

$U_{\text{Meso, Eutro}}$ = 22.475 + .216 (SD) -.086 (COND) + 3.133 (TON)
+31.434 (TP4) + 3.311 (CI) -.100 (COD)
- .064 (TS) + .080 (PP) - .687 (CHA)

Table VII continued

CHARACTERISTICS OF LAKES CLASSIFIED:

	SD	COND	TON	TP4	CI	COD	TS	PP	CHA
Lake Ten a (Oligo II)	3.5	33.0	1.07	.080	4.75	35.0	126.0	21.82	12.18
Lake Twelve A (Oligo I)	11.0	51.0	.28	.014	1.06	10.0	35.0	.24	.93

APPENDIX A

December, 1968 Trophic Characteristics Data Used In Cluster, Canonical, Factor and Discriminant Analyses Examples

Trophic Criteria

Lake Number	Secchi Disc	Productivity	Organic Nitrogen	Total Phosphate	Cation Index	Chemical Oxygen Demand	Total Solids	Primary Productivity	Chlorophyll a
1	8.0	48.0	.440	.110	3.00	16.0	158.0	13.45	5.56
2	4.3	50.0	.570	.100	2.96	31.0	96.0	6.56	3.28
3	7.0	39.0	.650	.040	4.00	21.0	66.0	7.52	6.36
4	5.7	49.0	.620	.090	2.47	22.0	161.0	10.28	4.84
5	4.0	63.0	1.120	.160	2.18	87.0	85.0	87.00	21.91
6	3.0	38.0	.720	.110	2.03	48.0	180.0	.58	2.98
7	6.0	33.0	.590	.050	3.79	14.0	120.0	.33	3.32
8	4.0	185.0	1.220	.080	.33	37.0	166.0	55.45	19.44
9	3.3	54.0	.890	.040	2.29	68.0	91.0	12.70	4.11
10	5.0	44.0	.810	.100	2.68	48.0	102.0	17.40	3.29
11	6.0	43.0	.790	.043	1.93	45.8	98.0	12.91	3.14
12	4.0	53.0	.410	.120	5.00	26.0	72.0	4.26	4.28
13	2.0	40.0	.580	.016	3.00	20.4	86.0	.18	1.81
14	2.5	90.0	1.170	.050	.80	55.0	153.0	35.57	12.49
15	2.0	67.0	1.090	.050	1.10	56.0	122.0	43.01	9.68
16	2.0	22.0	.860	.080	4.57	72.0	85.0	3.36	3.34
17	2.0	60.0	1.130	.050	1.85	79.0	141.0	53.55	8.62

Lake Number	Secchi Disc	Productivity	Organic Nitrogen	Total Phosphate	Cation Index	Chemical Oxygen Demand	Total Solids	Primary Productivity	Chlorophyll a
18	3.3	45.0	.42	.080	3.24	31.0	85.0	7.46	24.16
19	2.0	41.0	1.55	.160	1.50	64.0	95.0	10.45	10.76
20	2.5	275.0	1.33	.250	.39	50.0	258.0	234.70	38.98
21	4.5	86.0	.83	.060	.80	40.0	96.0	3.59	5.51
22	6.0	533.0	.33	.590	.21	12.0	371.0	.05	1.60
23	3.0	263.0	1.19	.320	.31	65.0	234.0	77.54	11.02
24	2.8	106.0	1.38	.170	.69	44.0	115.0	69.11	11.39
25	6.0	106.0	.73	.060	.16	17.0	160.0	6.06	.18
26	7.0	38.0	.74	.120	4.60	29.0	122.0	2.09	6.86
27	6.0	43.0	.56	.200	1.91	28.0	86.0	41.02	12.70
28	2.0	166.0	3.84	.420	.56	78.0	263.0	26.91	9.48
29	5.0	32.0	.86	.050	4.70	23.0	40.0	5.33	6.44
30	5.0	16.0	.89	.040	2.89	38.0	90.0	1.42	2.56
31	3.0	67.0	1.87	.360	.73	66.0	166.0	54.35	21.40
32	3.0	59.0	1.71	.150	1.45	25.0	115.0	124.30	30.13
33	4.0	55.0	.85	.280	1.09	52.0	83.0	12.23	5.76
34	1.0	307.3	3.81	.239	.45	113.0	227.3	386.00	34.10
35	1.0	332.7	5.31	.391	.48	157.0	226.3	1020.00	72.10
36	2.5	216.7	1.76	.046	.36	48.3	127.3	37.00	17.70
37	2.0	274.7	4.10	.222	.45	111.7	183.3	274.00	33.10
38	1.5	286.0	3.00	.199	.42	78.3	185.3	183.00	45.40
39	4.0	136.0	.97	.021	3.67	25.3	35.3	11.00	8.40

Lake Number	Secchi Disc	Productivity	Organic Nitrogen	Total Phosphate	Cation Index	Chemical Oxygen Demand	Total Solids	Primary Productivity	Chlorophyll a
40	20.0	56.0	.34	.010	1.26	12.8	38.3	1.87	1.20
41	11.0	22.3	.11	.010	5.48	1.6	53.0	.48	1.28
42	11.0	22.5	.21	.013	5.59	2.8	54.5	1.17	3.18
43	12.0	29.7	.23	.019	3.10	7.4	83.3	.01	1.19
44	11.0	49.3	.33	.019	2.88	6.8	20.0	.03	1.07
45	9.0	46.7	.19	.016	3.65	2.5	15.0	.28	2.31
46	5.0	41.0	1.07	.031	3.25	54.0	84.0	1.97	3.82
47	10.0	30.0	.18	.019	2.85	3.0	36.5	.53	2.16
48	2.0	37.3	.84	.033	2.44	61.2	19.3	4.80	1.64
49	18.5	40.3	.23	.008	2.23	10.0	20.0	.66	.81
50	10.0	40.0	.28	.013	2.85	15.0	30.0	3.76	6.85
51	3.0	36.0	.88	.037	2.37	55.9	81.3	3.74	2.70
52	16.0	48.7	.20	.010	3.02	2.5	42.0	.98	1.58
53	15.0	36.3	.31	.015	3.42	4.6	52.7	1.99	1.30
54	9.0	54.0	.21	.016	2.87	1.5	31.7	.91	2.13
55	11.0	41.3	.38	.018	1.99	3.1	40.0	.41	1.20

All units are mg/L except Secchi Disc (ft), Conductivity (umho cm^{-1}), Cation Ratio, (dimensionless) Primary Production (mg-C/hr/m^3), and Chlorophyll a (mg/m^3)

MULTIVARIATE TECHNIQUES FOR THE CLASSIFICATION OF LAKES AND THE STUDY OF EUTROPHICATION

Summary Statement

by

The Panel Members

MULTIVARIATE TECHNIQUES FOR THE CLASSIFICATION OF LAKES AND THE STUDY OF EUTROPHICATION

The members of the panel, although having diverse interests and points of view have managed to agree on some board summary statements. These statements concerning the usefulness of multivariate techniques in lake classification and the study of eutrophication may be summarized pointwise as:

(1) As no universal lake model exists now, or will exist in the foreseeable future, a use of a multivariate trophic classification system lies in its ability to tell us what one of the available lake models we should use for a particular trophic situation.

(2) A trophic classification system by supplying the investigator with a reasonably reliable estimate of a lake's trophic state provides experimental design guide lines for further study of the lake.

(3) Some multivariate procedures (e.g. factor analysis) are technically underdeveloped and accordingly should be used with discretion.

(4) The multivariate technique of principal component analysis offers the researcher with a method of determining, in a statistical sense redundant variables in the complex of variables describing trophic state.

(5) There appears to be considerable interest in developing a trophic state index. The most logical approach to establishing such an index is the use of multivariate procedures such as discriminant and cluster analysis.

(6) Classification procedures offer a method of classifying lakes not only with respect to trophic state but with respect to other use objectives. For example, qualitative ideas such as "Fishability", "Swimability", Boatability", etc., could be quantified by considering the appropriate variables describing each. It is suggested that the use of such terms would be desirable in public relations and because of their quantitative nature could be incorporated into management model objective functions.

(7) Multivariale techniques such as cononial analysis offer a method for establishing correlative relationships between two sets of variables (e.g. trophic state characteristics and lake basin parameters influencing eutrophication).

In conclusion it should be pointed out that multivariale procedures are statistical procedures and like all statistical procedures, interpretation of their results in terms of cause and effect must be done with caution. However, these techniques do provide the researcher with valuable tools for making inferences and predictions concerning the lacustrine ecosystem.

MULTIVARIATE TECHNIQUES IN LIMNOLOGY

by

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MULTIVARIATE TECHNIQUES IN LIMNOLOGY

INTRODUCTION

The classification problem in limnology is no different in principle from the problems of biological systematics or library science, and the remarks of Gilmour and Walters (1964) are pertinent to any of these fields. Multivariate techniques do little to resolve the philosophical problems inherent in classificatory work. Indeed, if all the thinking is done by the computer and the output accepted as revealed truth, multivariate techniques are no more than a complex but rapid method of attaining a dubious synthesis of questionable interpretation. This statement is not intended as a total condemnation of multivariate statistics in limnology or any other field. The limnologist or engineer can make good use of multivariate methods if he understands the limitations and idiosyncrasies of the various techniques.

THE BASIS OF MULTIVARIATE CLASSIFICATIONS

Any water body can be described by an infinite set of attributes -- morphometric, chemical, esthetic, economic and biological. Ignoring, for the moment, our criteria for the relevance and importance of characteristics, we may arrange the information in a lake x attribute matrix:

$$\begin{array}{ll|l} & & \text{lakes} \\ & & 1 \;\ldots\;\ldots\;\ldots\;\ldots\;\ldots\; m \\ \hline & 1 & \\ & \vdots & \\ \text{attributes} & \vdots & \\ & \vdots & \\ & n & \end{array}$$

in which the attributes may be binary, ranked or continuous variables. Given such a matrix we may wish to group either the attributes or the lakes. Principal components reduction, canonical analysis and factor analysis (Shannon, this report) are appropriate techniques for the study of attribute groups and will not be discussed here. The first step in classifying or grouping the lakes is the construction of an m x m matrix of similarities or differences.

$$
\begin{array}{cc|cccccc}
 & & & & \text{lakes} & & & \\
 & & 1 & \ldots & . & . & . & ..\,m \\
\hline
 & 1 & & & & & & \\
 & . & & & & & & \\
 & . & & & & & & \\
\text{lakes} & . & & & & & & \\
 & . & & & & & & \\
 & . & & & & & & \\
 & m & & & & & &
\end{array}
$$

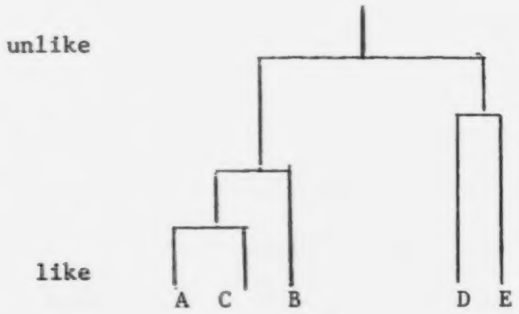
A multitude of similarity and difference measures have been proposed (Sokal and Sneath, 1963). Correlation and Euclidean distance are commonly used and, for binary data, various contingency measures. This is not the place for a critical discussion of all possible similarity indices, but it should be noted that "shape" coefficients (correlation) and "size" measures (Euclidean distance) lead to different results. For binary data, measures which consider negative matches as evidence of similarity (correlation) differ from those which do not (per cent similarity). Thus we are faced with a broad choice of alternatives at an early state of the classification process. Numerical classifications are not unique and the user must make a choice from an array of similarity methods and grouping strategies.

METHODS

Shannon (this report) has discussed a number of useful multivariate techniques. Two additional methods which have been used in ecology and other fields are hierarchic methods and the ordination procedure of Gower (1966).

Lance and Williams (1967) have reviewed a number of hierarchic methods. Many variations are possible but the following exemplifies one general method.

Given individuals (lakes) A, B, C, D and E characterized by multiple attributes, compute a similarity matrix. Scan the matrix and make a decision of the type: A and C are more alike than any other pair. Form the group AC, compute its average characteristics over all attributes and compute the similarity of AC with all individuals or groups remaining. Repeat the process until all individuals have been incorporated. The end product can be displayed as a dendrogram.



The major advantages of hierarchic methods are their simplicity and computational ease. Drawbacks include the tendency to generate residual groups and the distortion inherent in reducing a multidimensional situation to a two-dimensional tree. Like a decorative "mobile", the entire tree is free to pivot at each intersection, so the dendrogram does not indicate that A or C is closer to B. Individual fusions do not use the information in the similarity matrix very efficiently. Certain strategies may generate "reversals" in which groups fuse at a lower level than did their constituent individuals. In spite of these drawbacks, hierarchic schemes are useful for rapid sorting of date and may be the only feasible method for large arrays such as a series of plankton samples (Brown, 1969).

The ordination method developed by Gower (1966) has much in common with principal components reductions and other standard techniques. The basic similarity matrix may be of the correlation or distance type. Relationships are projected on a series of orthogonal axes and the original inter-individual distances are preserved. Ordination retains the original information much better than do the hierarchic methods. However, storage requirements and running time are greatly increased. In some cases the added information may not warrant the effort involved. (Webb *et al*, 1967) have compared methods applicable to certain types of ecological data. A related paper (Williams *et al*, 1969) is of special interest since it deals with multivariate analysis of successional states analogous to eutrophication.

FLIES IN THE OINTMENT

It is obvious that there is no single multivariate method for classifying lakes, watersheds or plankton samples. Various combinations of measures and grouping techniques may be appropriate for different kinds of data and particular problems. Selection of a technique is largely a matter of informed judgment and experience with a variety of methods.

The real problem in multivariate classification is not the method but the original data. All such techniques are designed to minimize within-group variances or distances and maximize those between groups. This dependence on the variance requires that the attributes be given in some standardized units. Transformations such as the logarithmic will alter the variances and the classification. The most serious problem is redundancy of characters. Suites of redundant attributes can dominate the analysis. The problem lies in distinguishing between attributes which have been measured several times under different names e.g. lake "size" as area, mean depth, maximum depth, and those correlated by some important causal mechanism e.g. phosphorus, chlorophyll. Principal components reduction of the attributes identifies such correlated characters but does not tell one what to do with them. Factor analysis (rotation) increases the problem. In neither case are the eigenvalues a valid measure of importance. Presence-absence data for plant and animal species are free of the redundancy problem since there is a logical dividing line between the attributes and different lakes with many species in common can be considered similar. Even here problems may arise. If the species are members of groups of different size e.g. diatoms and copepods, the resultant classification will be dominated by diatoms and tell little about copepods unless the two groups are highly concordant.

Hierarchic techniques will usually isolate rare but unusual individuals in the residual group. In ordination procedures, relative abundance of individuals in confounded with the degree of difference and a rare but peculiar type may not appear in the first few vectors. Thus a single eutrophic or transitional lake in a sample of oligotrophic lakes might not be detected.

The classical multivariate techniques, including the discriminant function, assume that individuals not included in the original analysis, are part of the same statistical universe. In limnology we very rarely sample, or even define, the universe under consideration. For this reason, I feel it is much safer to regard the sample as a closed system. Multivariate methods can be used to portray relationships within the sample but inferences about the rest of the world should be cautious ones.

USES OF MULTIVARIATE TECHNIQUES

Multivariate techniques are useful in processing and simplifying large quantities of information. The "arbitrary" nature of the sorting rules may generate groups and relationships which the more flexible human mind might never detect.

Multivariate classifications can be used in experimental design and analysis. There would seem to be some virtue in analyzing causal mechanisms in systems where some variables are constant in fact rather than adjusted by regression techniques.

A "good" classification is one which allows predictions to be made about variables not used in its construction. Unique or valuable lakes will be studied and managed on an individual basis but other cases such as the 15,000 lakes of Wisconsin will require some extrapolation of diagnostic and management techniques. Perhaps this is the area where miltivariate techniques will have their greatest impact.

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THE CHEMOSTAT
A CONTINUOUS FLOW ALGAL ASSAY

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THE CHEMOSTAT A CONTINUOUS FLOW ALGAL ASSAY

INTRODUCTION

During the last few years the use of continuous cultivation of organisms has become very popular. However, continuous cultures are not new; in fact, they were used back before 1900 to produce vinegar and yeast and to treat municipal wastes. One of the first laboratory scale units was set up by Ketchum and Redfield in 1938 (see Malek and Fencl, 1966) in which diatoms were grown in artifically illuminated tanks. The first real understanding of the fundamental relations in continuous cultures became known when the papers of Monod (1949) and Novick and Szilard (1950) were published. Since then many aspects of algal growth and physiology have been studied using continuous culture systems. The effect of high intensity light upon the growth of chlorella was studied (Matthern, et al, 1969). Wassink, et al, (see Burlew, 1953) showed that the efficiency of light-energy conversion in chlorella is 12 to 20 percent when the illumination is not too high, but decreases to 2 to 3 percent in full summer light. Pipes and Koutsoyannis (1962) observed that the population density of chlorella in light limited culture is directly proportional to detention time. Kok studied the effect of flashing light on chlorella growth while Tamiya, et al, observed the effect of diurnally intermittent light on chlorella growth (see Burlew, 1953). Temperature, light, and nutrient interactions during continuous culture were studied by Maddox and Jones (1964). Bacterial cultures have been used to study the influence of growth conditions on cell wall composition (Tempest and Ellwood, 1969). Limiting nutrient studies on bacteria were investigated by Slezak and Sikyta (1967) and Tempest, et al, (1965, 1967). Studies of fungal growth and intermediary carbon metabolism under steady and non-steady state conditions were made by Carter and Bull (1969).

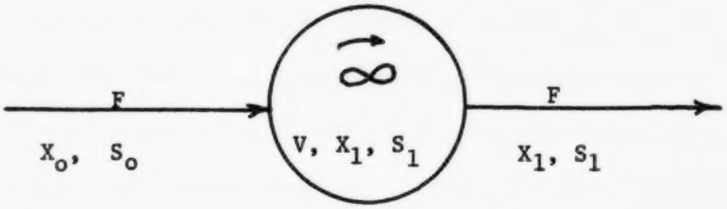
The current interest in continuous-flow steady state systems stems from the need for a laboratory method of evaluating biostimulatory response. Using continuous-flow systems it is possible to relate the growth rate to the limiting substance, determine yield and decay constants for a given substrate, and then estimate the standing crop of algae that will be supported by a given nutrient concentration. With the chemostat it is possible to determine the nutrient concentration which produces the maximum growth rate under any set of environmental conditions. However, the relationships between growth rate, nutrient concentrations, light intensity, photoperiod, temperature, and other factors must be determined. Once these relationships are determined, it should be possible to formulate mathematical models of algal productivity. The coefficients obtained from continuous flow cultures could be used to model a natural aquatic ecosystem using the kinetic equations derived from chemostat theory. However, Porcella (see Middlebrooks, et al. 1969) recommends against this; he believes that chemostat data should be used to gain useful information on productivity, nutrient cycling and successional relationships. Growth, decay, and yield coefficients derived from chemostat data could, however, be used as rate coefficients in sophisticated mathematical models. It should be emphasized that the chemostat is basically a highly simplified simulation of an aquatic ecosystem; thus, direct application of chemostat kinetics to complex natural ecosystems is not feasible.

A continuous flow system can be characterized as a constant volume system, being in a dynamic steady state with constant concentrations of all components, with all reactions proceeding at a constant rate. There are two general types of continuous systems: the turbidostat and the chemostat.

The turbidostat maintains a constant density by using a photocell adjusted to a specific light intensity; when the density of the culture reduces the light intensity, a valve opens allowing fresh medium to enter while an equal volume of the culture overflows into a sampling container. Up to about 1961, turbidostats were used more than chemostats, but due to their high cost and inconvenience, turbidostats are rarely used now.

Unlike the turbidostat, which is actually a semi-continuous system, the chemostat is a continuous flow system which maintains steady state by keeping a constant flow rate and volume. To maintain a constant volume, the flow in must equal the flow out. Furthermore, steady state is maintained only if the chemostat is completely mixed and kept in a constant environment, that is, constant temperature, light, etc. Figure 1 shows a model of a chemostat; the chemostat can be described as a Continuously Stirred Tank Reactor (CSTR) where the concentrations of substrate and organisms in the chemostat are equal to the effluent concentrations. By keeping the volume constant and adjusting the flow rate, any desired residence time can be maintained. Algal cells are added to the chemostat at a concentration X_0. After a certain time, the algal population reaches a steady state concentration X_1. The initial substrate concentration S_0 also reaches a steady state concentration S_1, where S_0-S_1 equals the amount of substrate utilized by the algal population X_1. Steady state is maintained when the cell growth rate equals the rate at which cells are washed out of the system.

The mathematical theory of continuous flow systems was derived from the typical microbiological growth curve obtained in batch cultures. Chemostats are usually run so that the algal population are in the


F
F
X_o, S_o
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X_1, S_1

FIGURE 1. SINGLE STAGE STIRRED REACTOR (CSTR)(From: Provisional Algal Assay Prod.)

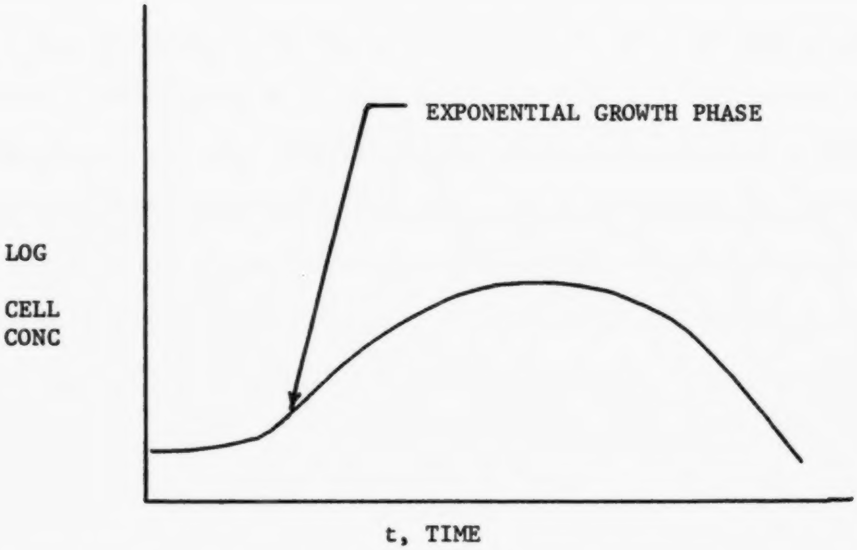


FIGURE 2. GROWTH CURVE FOR A BATCH REACTOR (From: Provisional Algal Assay Proc.)

exponential growth phase (see Figure 2). In this phase, the algal growth rate is a maximum and balanced growth is achieved. With balanced growth, all nutrients are present in excess and development is not inhibited by products of metabolism. Continuous culture allows the algae to grow in the exponential growth phase where they are permanently "physiologically young" (Malek and Fencl, 1966). The main advantage of continuous cultivation is the steady state condition that evolves. Cultivation can proceed for as long as desired or can be changed simply by regulating the nutrient flow rate.

The chemostat apparatus need not be complicated; Figure 3 shows a simple chemostat proposed by The Joint Government-Industry Eutrophication Committee on Methods (Pearson, et al., 1968). Operation consists of filling the chemostat with the test substrate, innoculating it with algal seed, and adjusting the flow rate so as to give the desired residence time. Environmental conditions such as temperature, light, intensity, and relative humidity must be kept constant. Daily observations of effluent volume, cell count in the reacter, pH, and conductivity should be made. At least twice each week the limiting substrate concentration and cell mass should be determined. For proper operation, the feed rate and the degree of mixing should be checked regularly. Using the data collected, the kinetic characteristics of the system can be modeled.

KINETICS

Kinetic equations for continuous algal cultures are possible because of the relationships between growth rate, the limiting nutrient, and growth yield. When these three relationships are combined using material balances, steady state kinetic equations can be formulated.

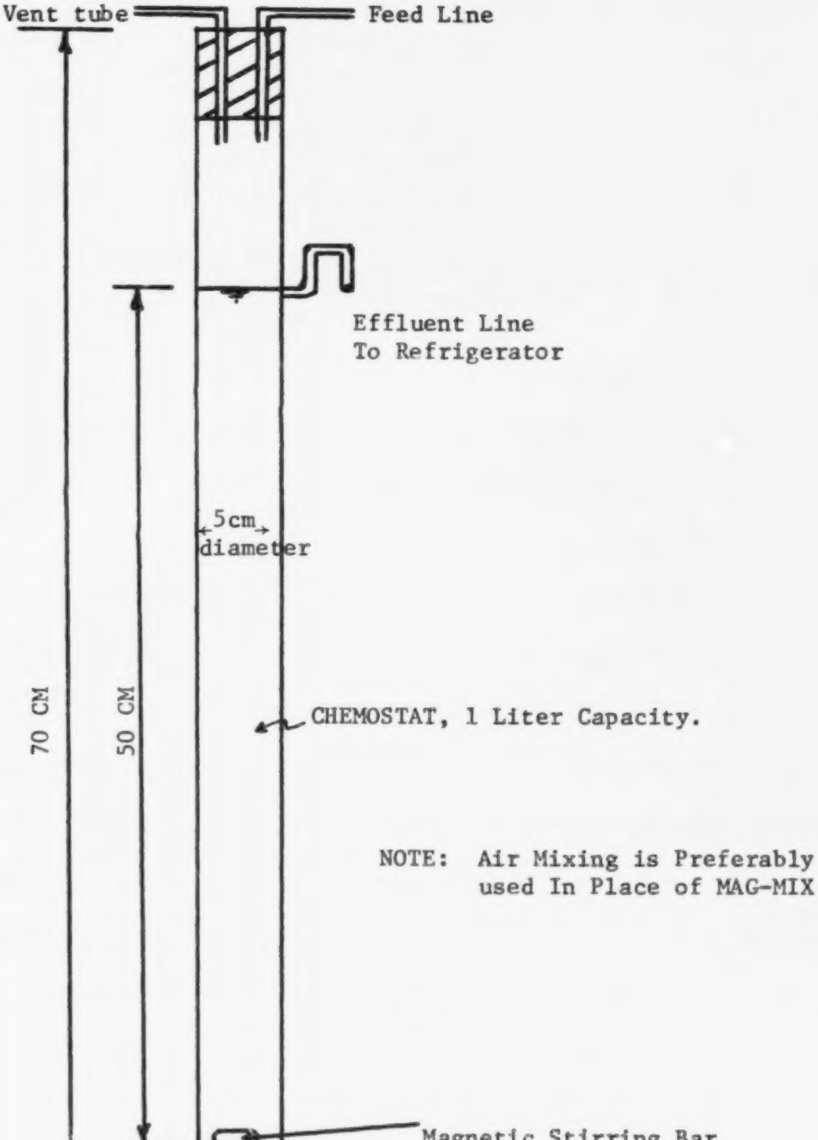

Vent tube
Feed Line
Effluent Line
To Refrigerator
5cm
diameter
70 CM
50 CM
CHEMOSTAT, 1 Liter Capacity.
NOTE: Air Mixing is Preferably used In Place of MAG-MIX
Magnetic Stirring Bar
MAG-MIX

FIG. 3 DETAILS OF PROPOSED CONTINUOUS CULTURE APPARATUS (CHEMOSTAT)
(From: Provisional Algal Assay Procedure)

The specific growth rate is a function of the limiting nutrient in the reactor. According to Monod (1950) and others, the relation between the specific growth rate and the limiting nutrient is expressed by the Michaelis-Menten equation:

$$(1) \qquad \mu = \hat{\mu} \left[\frac{S}{K_s + S} \right]$$

where

μ = specific growth rate, T^{-1}

S = growth limiting substrate concentration, ML^{-3}

K_s = half-saturation constant, ML^{-3}

$\hat{\mu}$ = maximum specific growth rate, T^{-1}

In 1913, Michaelis and Menten formulated their equation using the method of equilibrium rather than steady state. Today, however, enzyme kineticists use a Michaelis-Menten equation developed from steady state considerations. Both equations are identical except for the interpretation of the K_s values; the steady state K_s value includes dissociation of the enzyme-substrate complex while the equilibrium K_s value does not.

The maximum specific growth rate, $\hat{\mu}$, is the velocity of the reaction when the substrate concentration approaches infinity, that is, when all of the enzyme is combined with substrate, thus it is the maximum velocity obtainable for a specific enzyme at a given enzyme concentration. The half-saturation constant, K_s is characteristic of a particular enzyme and is equal to the substrate concentration at which the enzyme activity is half-maximum; K_s is also independent of the enzyme concentration.

Since it is assumed that growth rate is a function of the limiting substrate concentration, it seems reasonable to assume that the growth rate is also a function of the limiting substrate removal rate. However, yield coefficients (mass of cells produced per mass of substrate removed) for both algal and bacterial cultures have been shown to vary (Malek and Fencl, 1961).

A mathematical model for the chemostat is developed by taking material balances of cell mass and substrate. It is assumed that the Michaelis-Menten Equation adequately relates the growth rate to the limiting nutrient concentration, that a constant proportion of the cells are viable, and that the yield coefficient is constant. The following mathematical derivations are a condensation of those presented in the First Progress Report, Eutrophication of Surface Water - Lake Tahoe (McGauhey, et al., 1968).

Material Balance of Cells

$$\frac{dX_1}{dt} V = FX_0 - FX_1 + \mu X_1 V - K_d X_1 V$$

where

X_0 = influent cell mass, ML^{-3}

X_1 = effluent cell mass, ML^{-3}

F = flow rate L^3T^{-1}

V = reactor volume, L^3

k_d = decay coefficient, T^{-1}

At steady state,

$$\frac{dX_1}{dt} = 0$$

Therefore, assuming $X_0 = 0$,

$$(2) \quad \mu - K_d = \frac{F}{V} = \frac{1}{\theta} = \frac{1}{\theta c}$$

where

Θ = hydraulic residence time, T

Θc = mean cell age in reactor, T

Note that (μ-K_d) is the net growth rate. Substituting the Michaelis-Menten Equation into the above and solving for S_1, we get

$$(3) \quad S_1 = \frac{K_s(K_d + 1/\Theta)}{\mu - (K_d + 1/\Theta)}$$

For a particular microbial condition (μ, K_s, K_d), the effluent substrate concentration is a function of hydraulic residence time; that is, as the residence time increases, the effluent substrate decreases.

Material Balance of Substrate

$$\frac{dS_1}{dt} V = FS_0 - FS_1 - \left[\frac{VX_1 \mu}{Y}\right]$$

At steady state,

$$\frac{dS_1}{dt} = 0$$

Therefore

$$(4) \quad X_1 = \frac{Y(S_0 - S_1)}{\mu\Theta}$$

Since we assumed a constant yield, we can relate growth rate to substrate removal velocity.

$$(5) \quad \mu = Yq$$

where

Y = yeild coefficient = $\frac{\text{mass of cells produced}}{\text{mass of limiting substrate removed}}$

q = substrate removal velocity, T^{-1}

Substituting Equation (5) into Equation (2), we get

$$(6)\quad \frac{1}{\Theta} = Yq - K_d$$

To evaluate the kinetic growth characteristics, $\hat{\mu}$, K_s, Y, and K_d, equations (1) and (6) are solved graphically as shown in figures 4 and 5.

Data for the above analysis can be obtained by running a series of three steady state experiments for each nutrient and organism. For a given nutrient and organism, three different residence times or feed concentrations should be used, resulting in a different effluent concentration, S_1. Using the system constants ($\hat{\mu}$, K_s, Y, and K_d), the specific growth rate for any steady state nutrient level can be predicted. Thus,

$$\mu = \frac{\hat{\mu}\, S_1}{K_s + S_1} = \text{specific growth rate}$$

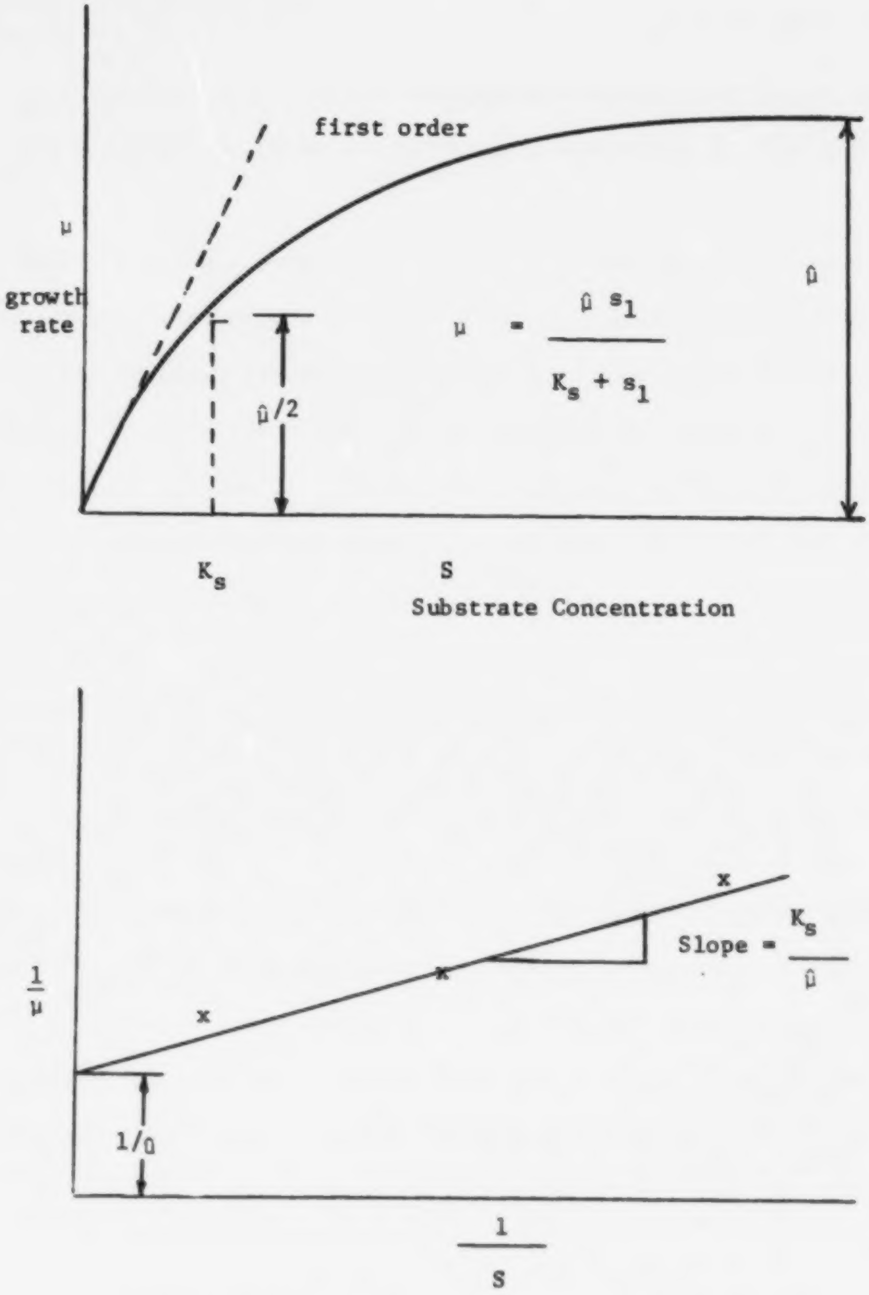
The net growth rate of the system can then be determined using Equation (2),

$$\mu - K_d = \frac{1}{\Theta c} = \text{net growth rate}$$

Other kinetic equations have been derived to calculate cell concentrations in continuous cultures. For example,Kono (1968, 1969) notes that predicted cell concentrations using the Monod equation do not fully correspond to observed values in continuous cultures. He derived a rate equation based on critical cell concentration and found good agreement between predicted and observed cell concentrations.

CHEMOSTAT VERSUS BATCH CULTURE

Today, much research is being done on determination of growth rates and nutrient uptake rates. Most of this work is analyzed according to



Straight line form. $\frac{1}{\mu} = \frac{K_s}{\hat{\mu}} \cdot \frac{1}{s} + \frac{1}{\hat{\mu}}$

FIGURE 4 MICHAELIS-MENTEN KINETIC MODEL

(From: Provisional Algal Assay Procedure)

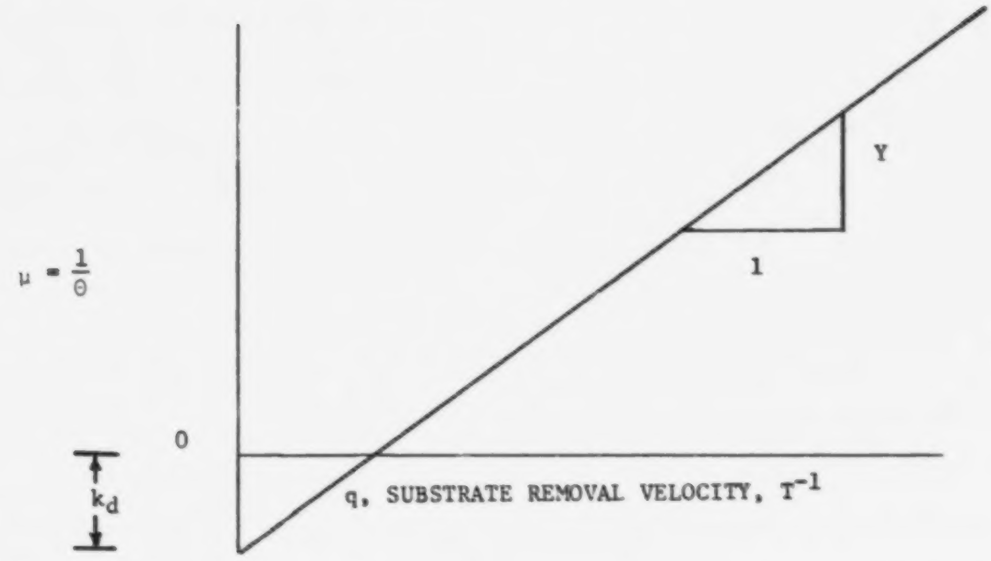


FIGURE 5 PLOT OF CELL CONTINUITY EQUATION,

$1/\Theta = Y q - k_d$

(From:Provisional Algal Assay Procedure)

Michaelis-Menten kinetics or some form thereof. However, some researchers work with batch culture methods while others use continuous flow cultures. And herein lies the problem: which method to use: Vaccaro (1969) used batch methods to study the response of natural populations from coastal areas to organic enrichment; Eppley, et al. (1969) used batch cultures to determine half-saturation constants for uptake of nitrate and ammonium by marine phytoplankton. Batch methods were used for bioassays for nitrogen and phosphorus with algae and aquatic weeds (Fitzgerald, 1969). The use of batch culture methods for kinetic studies is very popular and many other examples of batch culture research could be cited. We are faced with the situation of having two scientific camps, using two different methods to measure the same parameters.

As mentioned earlier, the chemostat is a dynamic, open system operating at steady state with the algal cells approximately in the same physiological state. The batch unit, however, is a static, closed system, with the physiological state varying with time as shown in Figure 2. Some might object to labeling the batch culture as a "static system" noting that the greatest changes in properties and number of organisms occur in batch culture. However, these changes are due to inhibition of the real dynamics of microbial growth by static conditions (Malek and Fencl, 1966). Fogg (1965) noted that it is difficult to study the effects of mineral nutrient limitation on exponential growth in batch systems since the effect of adding a low concentration of a particular nutrient is to shorten the duration of the exponential phase rather than reduce the relative growth rate. According to Fogg, exponential growth eventually stops in a batch culture because of the following factors: exhaustion of nutrients, decrease in carbon dioxide diffusion, pH change of the culture, light reduction by self-shading, and autoinhibition. The sensitivity and applicability of batch cultures in quantitating the growth characteristics of limiting nutrients have been questioned (McGauhey, _et al_., 1968).

Much more information is obtainable from the continuous culture method than from the batch method. One great advantage of the continuous system is that the standing crop level of organisms supported by a given nutrient concentration and residence time can be determined. The batch method merely relates the maximum growth rate to the initial substrate concentration; the substrate concentration at which the maximum growth rate occurs is not known. For continuous culture methods, once the kinetic constants are determined for a given organism and nutrient, the net or gross cellular gross rate attributed to different nutrient concentrations can be estimated. This is not true for batch culture methods.

The chemostat, however, is not a panacea; it has its disadvantages. Chemostat equipment is, in general, very complicated and expensive; it is often subject to breakdown resulting in the loss of steady-state conditions. Operation of the chemostat is complex and time consuming; it is not recommended for routine bioassays. Excessive growths of some algae on the walls of the chemostat is a common problem, especially with natural algal populations. Porcella (1969) reported that wall growths of diatoms and blue-green algae in a natural algal population were 3-30 times greater than measured in the effluent samples. He recommended that adequate mixing might be achieved through wall scraping or inhibition of those algae which become attached.

In the chemostat, as well as in the batch culture, the limiting nutrient may be added in excess of that needed for growth, resulting in a data point that has no relation to the growth rate. It has also been established that when the dilution rate varies, the limiting factor can be transferred from one nutrient to another (Malek and Fencl, 1962). When there are several limiting factors present, interpretation of results is very difficult (Porcella, 1969).

It has also been shown that the half-saturation value (K_s) usually decreases as the age of a continuous culture increases. This decrease is probably due to mutation and selection since the lower K_s will give the organism a strong competitive advantage (Malek and Fencl, 1962). However, mutation would only occur if the residence time was exceptionally long.

LAKE TAHOE STUDY

A comprehensive program of chemostat research is being carried on at Lake Tahoe (McGauhey, et al., 1968, 1969). In this study, both batch and continuous cultures were investigated to determine whether wastewater effluents contain materials which stimulate algal growth. Comparisons were made between the batch culture and chemostat to determine which technique to use for the best assessment of the biostimulatory properties of wastewater. It was concluded that the batch assay method is an economical tool for the assessment of the relative enrichment characteristics; that is, the batch assay is a useful but qualitative technique. Quantitative data required for ultimate control of eutrophication must relate algal growth rates to specific nutrient concentrations. Since only the chemostat gives this relationship, evaluation of chemostat data should allow quantitative estimation of the effect of specific nutrients on algal growth.

A comparison of batch and continuous flow assays was made with a nutrient concentration S_o (batch) equal to S_1 (steady state). From kinetic theory, the batch maximum growth rate should be much less than the steady state growth rate in the continuous flow culture. However, this was not observed, in fact, the batch growth rate was about four times greater than the steady state growth rate. Thus, it seems that the initial growth response of the batch culture is much greater than the growth rate maintained under steady state conditions. Measurement of the transient chemostat rate, that is, the growth rate before steady state was reached, showed it to be even greater than the batch rate. The credibility of the batch assay was questioned because of the unusual results obtained. It was concluded that further research was required to study the questionable characteristics of the batch assay and to verify the applicability of the chemostat in determining kinetic constants.

CONCLUSIONS

Like all laboratory techniques, the chemostat is an artificial system that does not consider variations in light intensity, temperature, and other environmental factors. It does not even consider photoperiod, although it has been shown that many algae respond differently to continuous light. The complex relationships between algae and substances in natural waters and associated bacteria are now known well if at all. However, batch cultures are also subject to these limitations.

Porcella (see Middlebrooks, et al., 1969) pointed out that environmental factors such as light, temperature, CO_2, and mixing are usually kept constant in order to eliminate their effect on the growth of algae.

Although these environmental factors may control productivity in nature, the growth rate can be related only to the limiting factors in the sample.

More research is needed to determine the effects of light, temperature, CO_2, and other environmental factors on algal growth characteristics. Additional work should be done on chemostats using natural algae populations. As we determine the effects of more environmental factors on growth characteristics, and as we switch to chemostats with natural algae populations, we come closer to simulating the natural aquatic ecosystem.

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RATIONAL MULTIVARIATE ALGAL GROWTH KINETICS

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RATIONAL MULTIVARIATE ALGAL GROWTH KINETICS

INTRODUCTION

In general, results are limited that show the relationship between growth rates and nutrient concentrations that can be applied to mathematical models of the eutrophication process. The application of these few results is further complicated because of the different assay techniques and environmental conditions employed by various laboratories. If such independently collected data are to be used, there should be some common base to which all results can be converted. Between various laboratories, two environmental parameters that most frequently differ in algal assay results are light intensity and temperature. An attempt is made in the following sections to modify the most commonly used expression to describe algal growth kinetics to correct for temperature and light intensity.

KINETICS OF ALGAL GROWTH

The formulation most frequently used to show the relationship between growth rate and nutrient concentration in algal assays is the Michaelis-Menten (1913), or Monod (1949), expression

$$\mu(S) = \frac{\hat{\mu}(S)\, S}{K_s + S} \qquad \text{[light constant]} \qquad (1)$$

where

$\mu(S)$ = specific growth rate $\left(\frac{\text{gm cells produced}}{\text{gm cells-day}}\right)$, time^{-1}

$\hat{\mu}(S)$ = maximum specific growth rate as a function of nutrient concentration, time^{-1}

K_s = nutrient concentration at one-half the maximum specific growth rate, mass per unit volume

S = rate limiting nutrient concentration, mass per unit volume.

The maximum growth rate and the K_s value have been reported by Monod (1950, 1942) to be constants for a particular nutrient and culture. This expression has been used very successfully, but it must be remembered that it relates the effect of a single nutrient on the growth rate. In situations involving complex media and mixed cultures, the expression may apply only over limited ranges and many environmental factors also may influence the growth rate.

LIGHT INTENSITY AND ALGAL GROWTH RATE

It has been shown by Shelef _et al_., (1968) that the relationship between light intensity and the specific growth rate of algae is closely approximated by a hyperbolic function similar to the Monod equation above. That is

$$\mu(\ell) = \frac{\hat{\mu}(\ell)\ \ell}{K_\ell + \ell} \qquad \text{[nutrient constant]} \tag{2}$$

where

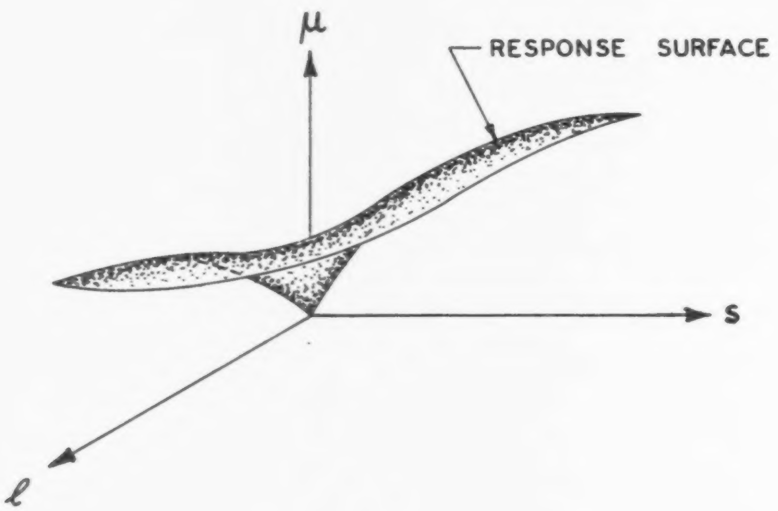
$\mu(\ell)$= specific growth rate for given nutrient concentration, time^{-1}

$\hat{\mu}(\ell)$= maximum specific growth rate attainable at given nutrient concentration, time^{-1}

ℓ = solar radiation or light intensity, langleys or calories/cm^2

K_ℓ = light intensity at one-half the maximum specific growth rate attainable (k), calories/cm^2.

Therefore, $\hat{\mu}(S,\ell)$ equals the maximum growth rate when S is a maximum and ℓ is optimum, or as shown below the growth rate is a function of the response surface.



Now, perhaps;

$$\mu(S,\ell) = \hat{\mu}(S,\ \ell)\ \frac{S}{K_s + S} \cdot \frac{\ell}{K_\ell + \ell} \tag{3}$$

and when $S << K_s$ and $\ell << K_\ell$, the first order equation becomes,

$$\mu(S,\ell) \tilde{=} \frac{\hat{\mu}(S,\ell)}{K_s \cdot K_\ell} \cdot S \cdot \ell \ . \qquad (4)$$

Equation 4 becomes our "model" for testing.

TEMPERATURE AND ALGAL GROWTH RATE

Algal growth rate, as in all biological reactions, is a function of temperature. A simplified temperature function which applied to many chemical reactions and the biochemical oxygen demand (BOD) test may be applicable to algal growth. This is

$$\frac{\mu_1}{\mu_2} = Z^{(T_1 - T_2)} \qquad (5)$$

where μ_1 and μ_2 are the specific growth rates at temperature T_1 and T_2, respectively, and Z is the temperature coefficient (Phelps, 1944). Based upon results from independent studies as adopted by Streeter and Phelps (1925), the best mean value for Z when used in BOD relationships is 1.047. Because of the lack of temperature-growth rate data for algal systems, the value will be assumed to be 1.047 for the analyses reported herein.

MODIFICATION OF THE BASIC EQUATION

The temperature coefficient can be incorporated into the basic equation as follows:

and when $S << K_s$ and $\ell << K_\ell$, the first order equation becomes,

$$\mu(S,\ell) \overset{\sim}{=} \frac{\hat{\mu}(S,\ell)}{K_s \cdot K_\ell} \cdot S \cdot \ell \ . \tag{4}$$

Equation 4 becomes our "model" for testing.

TEMPERATURE AND ALGAL GROWTH RATE

Algal growth rate, as in all biological reactions, is a function of temperature. A simplified temperature function which applied to many chemical reactions and the biochemical oxygen demand (BOD) test may be applicable to algal growth. This is

$$\frac{\mu_1}{\mu_2} = Z^{(T_1 - T_2)} \tag{5}$$

where μ_1 and μ_2 are the specific growth rates at temperature T_1 and T_2, respectively, and Z is the temperature coefficient (Phelps, 1944). Based upon results from independent studies as adopted by Streeter and Phelps (1925), the best mean value for Z when used in BOD relationships is 1.047. Because of the lack of temperature-growth rate data for algal systems, the value will be assumed to be 1.047 for the analyses reported herein.

MODIFICATION OF THE BASIC EQUATION

The temperature coefficient can be incorporated into the basic equation as follows:

$$\mu(S,\ell,T) \overset{\sim}{=} \frac{\hat{\mu}(S,\ell,T_2\ )}{K_s \ . \ K_\ell} Z^{(T_1 - T_2)} S \ \ell \qquad (6)$$

where Z is about 1.047 and T_2 is 20°C.

Further modification is possible using Equation 1 and assuming $K_\ell << \ell$ in Equation 2 which yields

$$\mu\ (S,\ell) = \frac{\hat{\mu}(S,\ell)\ S}{K_\ell\ (K_s + S)}$$

$$\frac{1}{\mu(S,\ell)} = \frac{K_\ell\ K_s}{\hat{\mu}(S,\ell)\ S} + \frac{K_\ell\ S}{\hat{\mu}(S,\ell)\ S}$$

$$\frac{1}{\mu(S,\ell)} = \left(\frac{K_\ell\ \ K_s}{\hat{\mu}(S,\ell)}\right) \ \frac{1}{S} + \left(\frac{K_\ell}{\hat{\mu}(S,\ell)}\right) \ .$$

Correcting $\mu(S,\ell)$ to 20°C yields $\mu(S,\ell,20) = \dfrac{\mu(S,\ell)}{Z^{T-20}}$.

Therefore,

$$\frac{Z^{T-20}}{\mu(S,\ell)} = \left(\frac{K_\ell\ \ K_s}{\hat{\mu}(S,\ell)}\right) \ \frac{1}{S} + \left(\frac{K_\ell}{\hat{\mu}(S,\ \ell)}\right)$$

$$\frac{Z^{T-20}}{\mu(S,\ell)} = \left(\frac{K_s}{\hat{\hat{\mu}}}\right) \cdot \frac{1}{S} + \frac{1}{\hat{\hat{\mu}}} \quad (\hat{\hat{\mu}} = \hat{\mu}/K_\ell) \ . \qquad (7)$$

The above mathematical model neglects many factors that can influence a biological system; however, it does incorporate two often neglected variables and it is possible that the equation will partially explain deviations between results from various investigators.

RESULTS AND DISCUSSION

To demonstrate compliance with, or deviation from, the theoretical model presented in Equation 7, it is necessary to calculate values for the "constants" K_s and $\hat{\mu}$. These values can be determined by plotting the values for the left-hand side of the equation as the ordinate and the reciprocal of the nutrient concentration as the abcissa.

Steady-state data obtained by McGauhey *et al*. (1969) are plotted in Figures 1 and 2. These data were obtained from chemostats with a capacity of $\sim$ 8,000 ℓ which were designed to simulate the shallow portions of Lake Tahoe. Lake Tahoe water used in the assays was pumped directly from the lake by means of a centrifugal pump located near the shoreline. The sample containing the enriching nutrient to be evaluated by assay was then pumped into the chemostats at a uniform rate appropriate to produce and maintain the desired concentration of sample in the LTW. Light intensity and temperature were monitored continuously.

Figure 1 shows the results obtained when the sum of the nitrate and ammonia nitrogen concentrations of secondary effluent from the South Tahoe Public Utilities District (STPUD) Wastewater Reclamation Plant was assumed to be the limiting nutrient. Other forms of nitrogen and the phosphorus concentrations were plotted for the secondary effluent sample, but all were poorly correlated which indicates that the nitrate and ammonia nitrogen were limiting the growth.

Figure 2 shows the results obtained when the orthophosphate concentra-

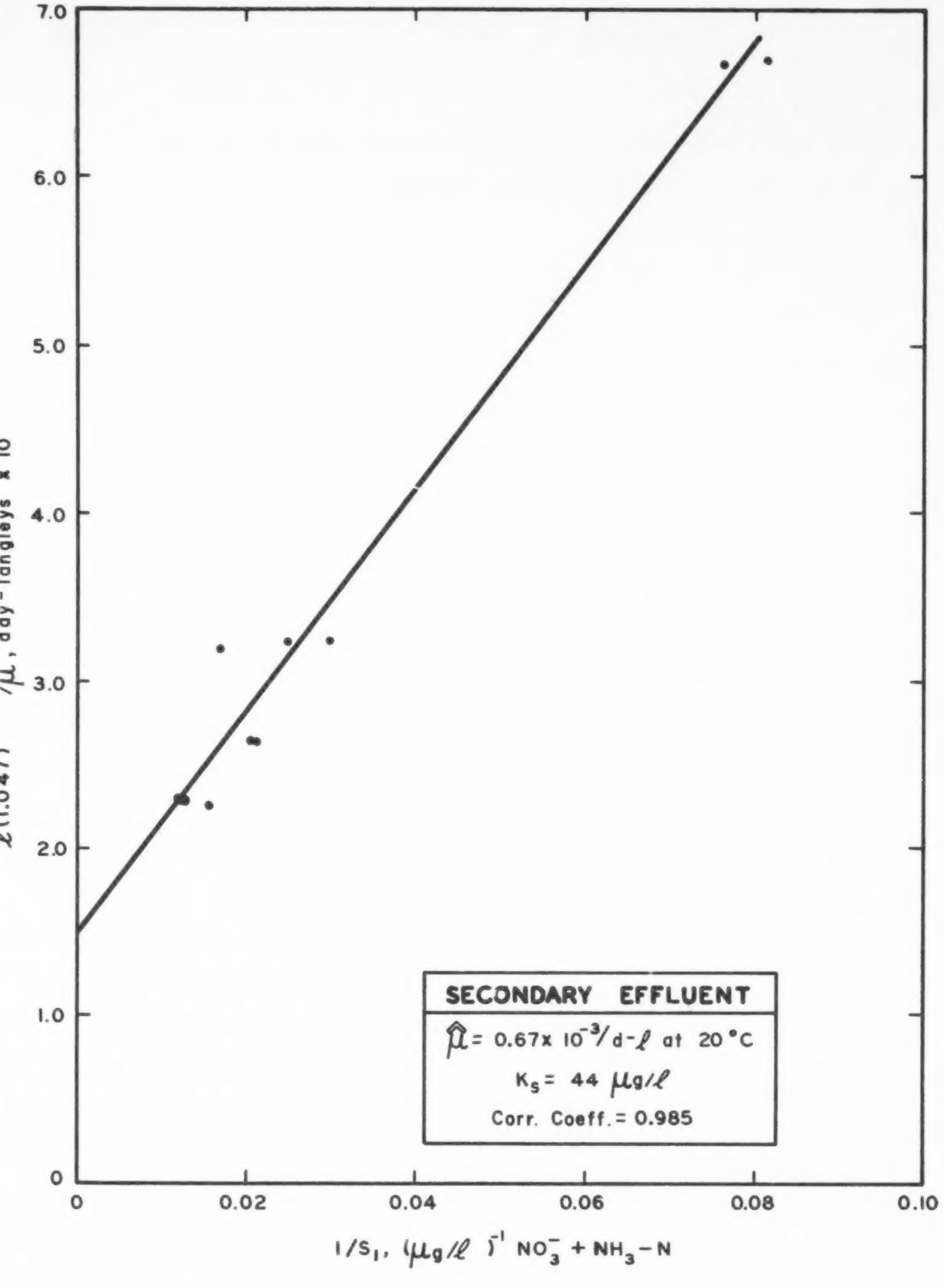


FIGURE 1. PLOT OF EQUATION (7) ASSUMING THAT THE SUM OF THE NITRATE AND AMMONIA NITROGEN CONCENTRATIONS REPRESENT THE LIMITING NUTRIENT IN THE SECONDARY EFFLUENT

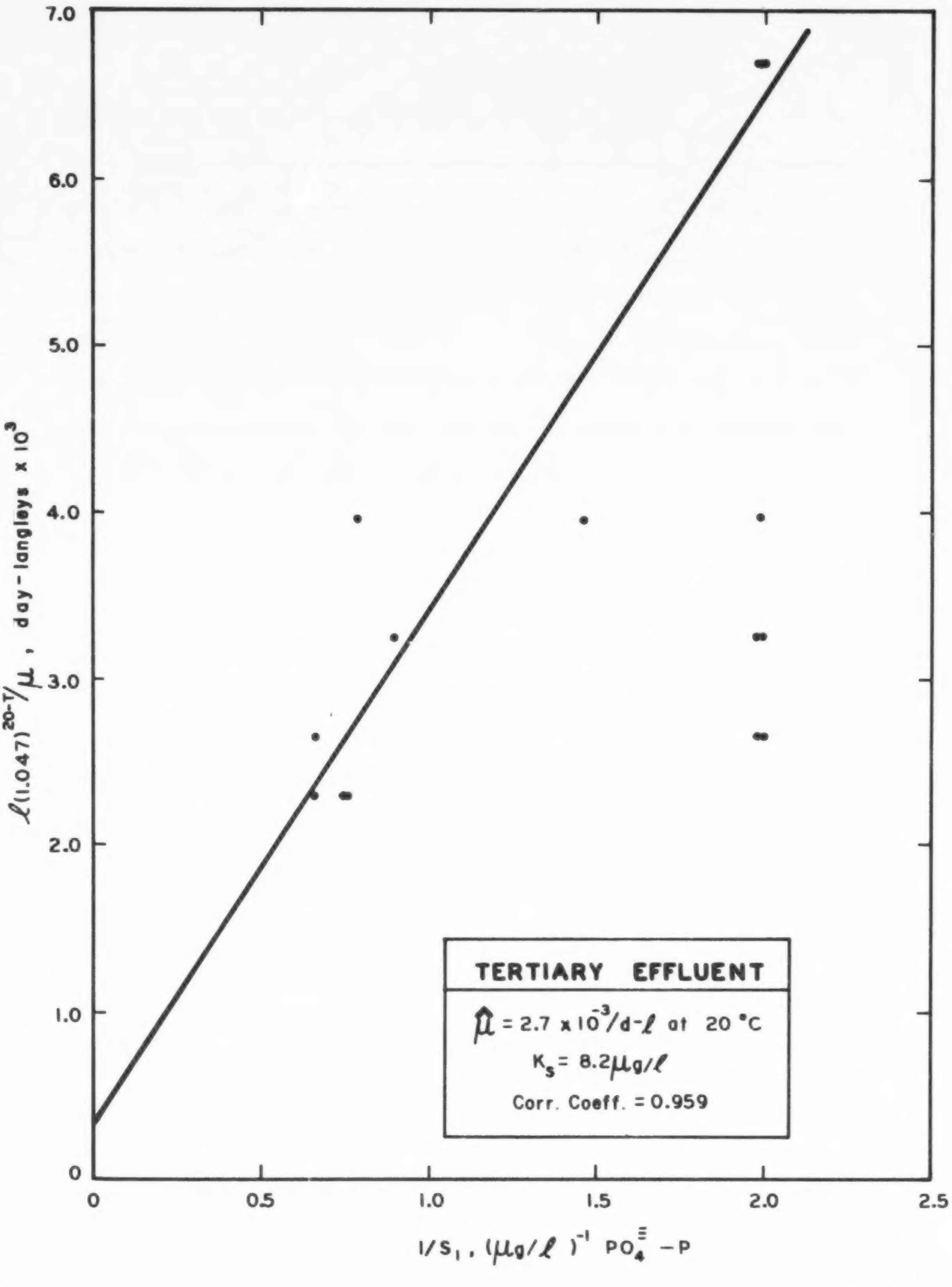


FIGURE 2. PLOT OF EQUATION (7) ASSUMING THAT THE ORTHOPHOSPHATE CONCENTRATION REPRESENTS THE LIMITING NUTRIENT IN TERTIARY EFFLUENT

tion (expressed as phosphorus) of tertiary effluent (Phosphorus removal only) from the STPUD plant was assumed to be the limiting nutrient. The regression analyses results shown on the figure do not include the five values for $1/S_1$ that equal 2.0 which fall well below the regression line. These were excluded because they represent the limits of the analytical procedure which was limited to 0.5μg/ℓ of orthophosphate as phosphorus. It is also very likely that the phosphate was exhausted long before the full potential for growth was achieved; thereby, resulting in a very low growth rate although the hydraulic residence time was adequate for a much greater response.

DISCUSSION AND CONCLUSIONS

The above results are only preliminary; however, it appears that the model describes data collected under natural environmental conditions even though the diurnal variation of light intensity and temperature were neglected and mean values for the period of steady-state operation were used in the equation. Additional data are being collected from various experiments performed at different temperatures and light intensities to determine if the model can be used to explain differences reported by various researchers. In view of the above results, it appears reasonable to expect the model to compensate for temperature and light intensity variations. Refinement of the temperature coefficient (Z) should further improve the model.

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NOTE: The presentation at St. Petersburg was made by Dr. Donald O'Conner, Department of Civil Engineering, Manhattan College. Dr. O'Conner's manuscript was not available for publication.

THE USE OF CHEMOSTATS
FOR THE STUDY OF ALGAL DYNAMICS

by

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THE USE OF CHEMOSTATS FOR THE STUDY OF ALGAL DYNAMICS

The use of chemostats for algal bioassay of nutrient elements is justifiable with reference to obtaining information relating to growth dynamics. However its routine use may not be justified due to its inherent difficulty of operation. In theory its advantages are based upon the concept of a steady state operation in comparison to the continuously varying environmental conditions of the batch assay, e.g. changes in nutrient concentration. The chemostat is claimed to be useful for assaying transient conditions because algal growth in the chemostat is operating under dynamic conditions at constant nutrient and cell concentrations. Because of the sensitivity of response to subtle changes in the environment, it theoretically provides a basis for assessment of abrupt or subtle changes in the environment.

The disadvantages of the chemostat relate primarily to the problems involved in operating it as a routine assay technique. Questionable results are generally obtained at very low nutrient concentrations equivalent to oligotrophic situations. These results in part may be attributable to fluctuations in flow rate as well as growth that may occur on influent lines and containers. Chemostats operating at low nutrient concentrations are functioning under difficult experimental conditions, since these concentrations are difficult to measure accurately. The maintenance of constant operating conditions in chemostats require almost constant attention as well appreciation of the theoretical basis of chemostat operation which may be generally beyond comprehension of the average technical staff. The equipment required to maintain constant operating conditions is generally complicated, subject to frequent failure and malfunction and generally leads to a high degree of frustration by the operating personnel since long runs may be wiped out due to failure at critical times. The operation of a chemostat is theoretically based on the uniform dispersion and suspension of the organisms throughout the media. If attachment organisms are present in the culture which grow on the wall of the chemostats, interpretation of data becomes most difficult and unreliable.

A chemostat in theory is basically a model of a highly simplified ecosystem. The Michaelis-Menton and Monad modifications of the growth of single cell organisms in suitable substrate concentrations, provides a theoretical basis for describing changes in cell numbers and mass. When these concepts are applied to the growth of algae in a chemostat, the growth response appears to follow particularly at high nutrient concentrations, such as would be found in the effluent of sewage treatment plants or in polluted streams or lakes. These high nutrient concentrations are those which permit short residence times in the chemostat probably no longer than a maximum five to seven days. The theoretical concepts do not, however, appear applicable to the growth of organisms at nutrient concentrations which require long residence times. Under these conditions the rate of multiplication of cells is so slow that washout and multiplication rate are essentially the same and no change in numbers or mass can be readily measured. The chemostat then becomes basically a batch test and the hydraulic complexity of the chemostat is not justified.

In developing a model of algal growth or response to environmental factors, it needs to be considered that the organisms in question are extracting from the environment both elemental materials, as well as molecular configurations which are then reassembled by biochemical processes into new cellular constituents which finally results in the situation where the cell divides forming two new cells. Thus, the simplest model to describe algal response to nutrient levels in the aquatic environment consists of A, representing the particular algal species; B, the elemental and molecular substances utilized in cell multiplication and the reaction of A + B giving rise to C which is cell number or mass. The reaction can be described as A + B = n + 1. The excess of 1 is that number or mass which is considered undesirable.

In this simplest of algal growth models unknown factors include both the nature of the species under consideration, the rate at which it uses specific elemental or molecular substances and that number or mass of the particular species which is considered undesirable. At the elemental level of nutrient requirement, the quantities and state of oxidation of nitrogen and phosphorus is a variable which is still not clearly understood for the many species of algae that have been described as responding with high multiplication rates when certain critical nutrient levels have been reached or exceeded. The requirements for specific molecules, such as vitamins as a requirement for growth, is another unknown in the algal growth mechanism and particularly with reference to rates of growth which may lead to undesirable conditions. Thus, the ultimate choice of assay procedure, chemostat or batch, will depend on the nature of the response to be measured and which most accurately relates to the characteristics of the growth material.

DISCUSSION

MODELS OF ALGAL POPULATION DYNAMICS BASED ON THE CHEMOSTAT APPROACH

SANDERS: We are looking at one of the problems, most of these chemostats are not sterile. At least they start off sterile and they don't remain sterile over a period of weeks, so you have a fixed system, and one part of the system is not responsive.

WEISS: That is quite correct. The system we are trying to establish does require that the test water be preceded by fresh water. Most of the water we use has been treated by filtration.

WATTS: You have used the word, verify, and I think this is a good word to scrub from your vocabulary. I would suggest that the word you want to put in is adequacy, because if you ever want to verify a model, you really want to test its adequacy. Only by looking for lack of this (adequacy) do you start to learn. The other thing, I think biologists start to raise their eyebrows when anybody says, I had to change the parameter when I took this model over to another lake. This seems to suggest that they are saying this casts a lot of doubt upon the problem. I would say this is not the case, the thing is that when you have to start changing the structure of a model, then you are certainly casting doubt upon the model. When you have to change the parameter, these parameters are functions of other things and this is giving you information. We should look for why these parameters are having to be changed. These so-called constants in your equations, What are they actually constants of?

BELLA: Let's assume, for instance, that the algae did have a vertical distribution, against reality. That would mean they could grow vertically.

They could produce, simply because the only algae that contribute to the growth are the ones that are lightened.

O'CONNER: That is what I tried to get over. I knew it wasn't very adequate. The average over the depth. This is the growth.

BELLA: But if your algae are all located down at the bottom, your argument that your time scale allows you to better make the assumption of complete mixing. I don't think that argument is quite as valid here. If you didn't have that complete mixing the growth would be low regardless of your time scale.

MOREAU: Would you comment on how you arrived at this resolution of time in terms of weekly averages specifically in the case which we are looking at here?

O'CONNER: Specifically, in the case we are looking at here, we zeroed in immediately on how good it was. The question is, now that we have gone this far, why don't we go a little further and sample next year's? You could probably sample three times a week, but I think you would probably get a feedback from the modeling effort, saying that it doesn't agree with the data.

MOREAU: Could you comment on resolution of time in terms of other kinds of water quality considerations?

O'CONNER: In that respect, something like the surface elements, like detergents accumulating in the environment. You can deal with the lakes in a series of five tanks, and go through the same procedures.

PATTEN: I would like to get some clarification on mixing and growth relation.

O'CONNER: We took that equation and took it to the system two ways:

laterally and vertically, so now the model looks like top layer and bottom layer.

FRUH: Going back to the assay as a model in adequacy, is it part of your plan now that somebody should go back into the lakes and check whether your growth rate and production mass are similar to the lake itself?

WEISS: The original concept, the procedure, the PAAP test, was to use the batch test as an assay procedure for the establishing of nutrient levels in certain water. The contract was distributed around the country in order to evaluate local conditions. There were many kinds of problems involved in putting the chemostat together. In the original design, the chemostat and batch test were to be checked against the surface waters which were to be prepared by certain standard procedures we have been able to perform simple experiments funneling through membranes.

FRUH: Do these tests give you the nutrients?

WEISS: Your question was, and this is what is answered by the test itself, as far as this contract work, will the two tests give us essentially the same information and will one be easier to use than the other?

BARTSCH: Let me respond to your question, also. Obviously one of the key questions with respect to assaying procedures is whether or not and to what extent in judgment you apply this information to natural conditions. With respect to this particular effort, we would not have reached that stage of application yet, but we can point to some brilliant applications where the procedure has been applied in the field. Dr. Shapiro has done some work on an estuary in the Port of Huron. In connection with this program for the improved assay procedures, we contemplate that we will have reached an appropriate point when the major effort will shift over to determining how to apply this knowledge to the field. We can already foresee many difficult problems. One, for example, Case Three, we call it: How in the world do you take a sample from nature and say to it, Now what would you be like if we reduced the phosphorus content by fifty percent? How in the world do you do that without changing some of the other qualities of this sample? We have some ideas, but please tell me, write to me, call me collect - These procedures, which are provisional, are the same as the PAAP Test, except that we hope eventually to remove the first key, which is where it is official. These techniques are now delineated in a document which I would be willing to make available to any of you who intend to use it, and if you have that kind of desire, show some hands and I will put a pad up here and get your name and address.

EUTROPHICATION MANAGEMENT MODELING

A Position Paper

by

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EUTROPHICATION MANAGEMENT MODELING

INTRODUCTION

The workshop on modeling the eutrophication process will discuss attempts to model specific components of the eutrophication problem. The discussion of my panel will focus on management modeling.

Management models are used primarily as decision-making tools and their effectiveness can be measured according to their ability to provide an improved basis for making decisions. Little work has been done in eutrophication management modeling. However a significant amount of research has been done on water quality management modeling in river systems.

During the past three years I have developed management models for regional water allocation (Heaney, 1968), regional water pollution control (Heaney, _et_. _al_.), and air pollution control (Bunyard, _et_. _al_., 1970) using network-type mathematical programming formulations. The environmental system is described using a link-node configuration wherein the commodities of interest enter or leave the system at the nodes and move from node to node along feasible links. Economic optimization is achieved by establishing a criterion which measures the efficacy of moving along a given link. An upper and/or lower bound may be placed on flow along a link, i.e., a capacitated link. Also, amplified or attenuated flow along a link is permitted, i.e., generalized network.

This work indicates that environmental management models contain several common elements which are listed below.

1. statement of objectives
2. description of study area

3. selection of commodities to be analyzed
4. inventory of sources for each commodity
5. inventory of controls for each commodity
6. description of spatial and temporal transport processes
7. analysis of control costs and benefits
8. formulation of performance standards
9. formulation of institutional constraints
10. algorithms for system optimization

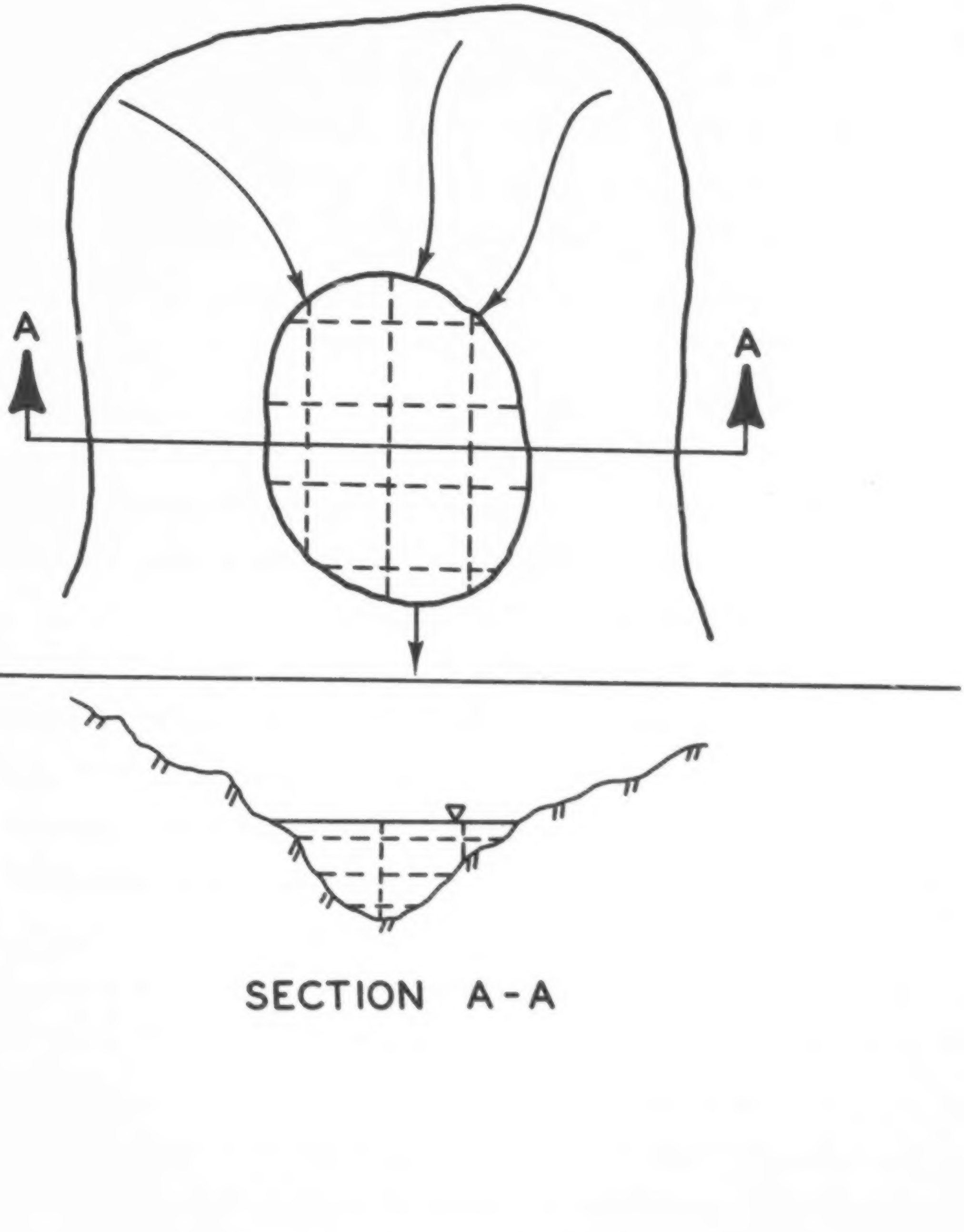
Each component will be described briefly in the balance of the paper. The integration of these components into a mathematical programming model will also be sketched. This presentation should provide a point of departure for our discussion on eutrophication management modeling.

1. Statement of Objectives

The logical first step in the analysis is to clearly and explicitly state the objectives of the study. It is incomplete to say that pollution should be minimized without placing a well specified list of constraints on the minimization. We might select as an objective the minimization of cost subject to satisfying a prespecified water quality goal. An alternative (but related) objective would be to maximize the water quality of the lake subject to a budgetary constraint.

2. Description of the Study Area

It is necessary to partition the entire area of interest into subsets which provide a mutually exclusive and collectively exhaustive representation of the system. The area of interest includes not only the lake itself but also its entire contributory drainage area. A sample partition of a lake into a number of cells is shown in Figure 1. Complete homogeneity is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SECTION A-A

FIGURE 1. SAMPLE DELINEATION OF STUDY AREA AND PARTITIONING LAKE INTO CELLS

assumed within a single cell. Also it is necessary to specify the time interval to be analyzed and how many such periods will be considered simultaneously. The average residence time of a parcel of water in the lake provides a guideline in selecting the total time horizon to be examined. It is not necessary that the total time horizon be divided into periods of equal duration although it is typical to select a uniform time interval such as an hour, day, or year.

3. Selection of Commodities to be Analyzed

It is often perplexing to attempt to select from the large subset of biological, chemical, and physical characteristics of pollution, some small subset of "indicator" commodities. It is not always apparent which commodities to select and much time can be wasted if clear selection guidelines are not established. Previous efforts have demonstrated that the commodities can be partitioned into three classes: carrier, conservative, and nonconservative (Heaney, 1968). This taxonomical scheme permits the investigator to view the model building process in a more generalized context.

4. Inventory of Sources for Each Commodity

During each time period it is necessary to describe how much of a given commodity is entering the system from the known sources, e.g., domestic and industrial wastes, farmland drainage. The pollutional load is estimated as the product of an activity coefficient, e.g. # BOD per capita per day, multiplied by the number of activity units, e.g., population. The activity coefficients are estimated by specialists in the environmental field whereas the estimates of activity units usually are obtained from demographic studies, Leontief-type models, and so on (Isard, 1960).

5. Description of Controls for Each Commodity

We need an inventory of the preventative controls for modifying the input quantity, and therapeutic controls for modifying the transport properties of the selected commodities. A tabular summary of such an inventory is shown in Figure 2. A single control may affect more than one commodity and more than one control may exist for regulating a given commodity.

6. Description of Temporal & Spatial Transport Processes

Given the selected commodities we may write continuity equations which describe the transport of a given commodity through the system, its interdependencies with other commodities, and its interperiod interdependencies with itself. These equations are a mathematical representation of the biological, chemical, and physical processes that are taking place in the system. A simple network representation of the spatial and temporal transport of a commodity i during two time periods is shown in Figure 3.

7. Economic Analysis

This phase of the discussion could include a cost analysis of the various controls and discussion of a methodology for quantifying benefits. One interesting question in developing eutrophication management models is the time lag that occurs from when an investment is made in a control and the water quality goal is achieved. In pollution of flowing bodies such as rivers and the atmosphere the goal is attained shortly after the control is implemented. This is not the case for "standing" bodies of water due to the residual effect. Thus, the time dimension must be added. The cost of attaining a given goal tends to increase as the time horizon

FIGURE 2. INVENTORY OF CONTROLS FOR EACH COMMODITY

	CONTROL						
COMMODITY	1	2	3	4	5	- -	p
x_1	x		x				
x_2		x					
.							
.							
.							
x_m					x		
- - -	- - -	- - -	- - -	- - -	- - -	- -	- - -
x_{m+1}				x			x
x_{m+2}	x	x					
.							
.							
.							
x_n	x	x	x	x	x		x

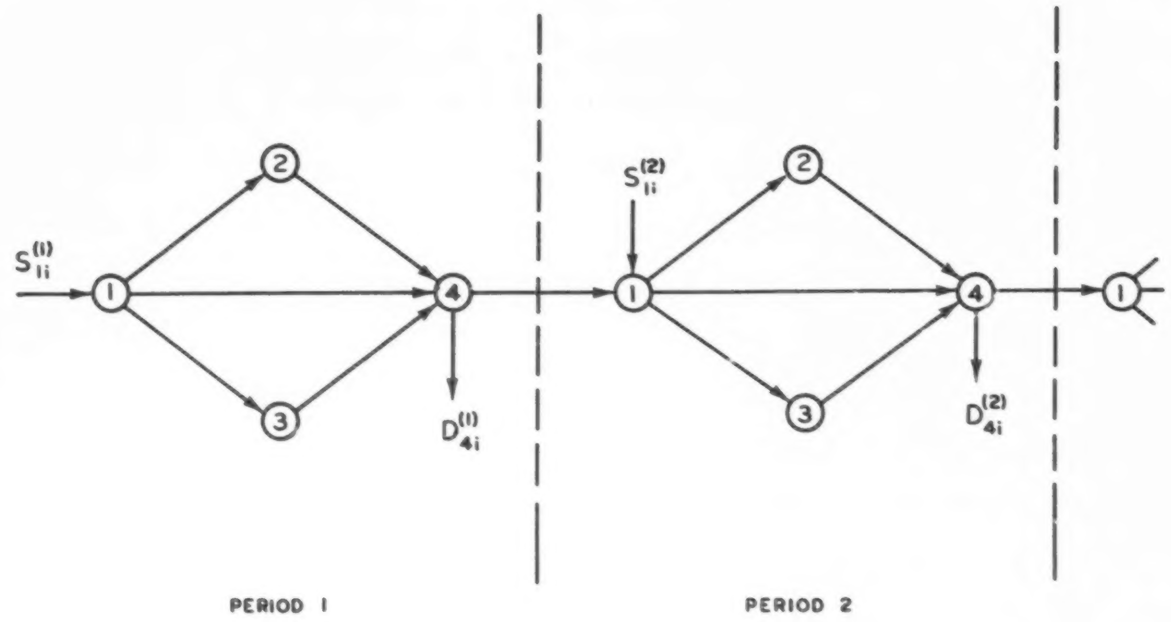

$S_{1i}^{(1)}$
1
2
3
4
$D_{4i}^{(1)}$
$S_{1i}^{(2)}$
1
2
3
4
$D_{4i}^{(2)}$
1
PERIOD 1
PERIOD 2

FIGURE 3. DESCRIPTION OF SPATIAL AND TEMPORAL TRANSPORT PROCESSES

is shortened.

The eutrophication investment problem might be characterized as a series of discrete investments over time which result in a continuous stream of benefits over an extended period of time. These benefits are both tangible and intangible. Our knowledge of the quantification of benefits is incomplete but it would include evaluation of recreation, land valued, esthetics, etc.

8. Formulation of Performance Standards

Environmental quality standards formulations fall into two broad categories: effluent or receiving system. The latter formulation requires a model which can simulate the movement of the pollutant through the carrying medium (air or water). A comprehensive evaluation of the allowable quantity of a given commodity at selected points in the lake is needed for each time period. Because of the extended residence time in lakes the standards need to take cognizance of the cumulative effects which occur.

9. Formulation of Institutional Constraints

Public institutions and policies provide the framework within which the activities of the water resource system function and hence exert a significant influence on the management of the system. The analysis and determination of optimal combinations of public institutions and policies that will promote fulfillment of environmental management objectives, together with periodic re-evaluation of the validity of the assumptions upon which they were originally based, are essential components of a sound management plan.

10. System Optimization

A significant advantage of using a network type formulation of environmental problems is that it permits us to utilize the wide variety of algorithms that are available for optimizing multi-commodity network flow problems. In addition there are the efficient mathematical programming formulations of transportation-type problems. A detailed discussion of optimization techniques is outside the scope of the discussion. The interested reader is referred to a recent book by Hillier and Lieberman (1967).

A schematic diagram of a mathematical programming management model for water and two carried commodities is shown in Figure 4. There is an objective function and a set of constraints. In this illustration the selected objective is to minimize the cost of satisfying specified water quality standards (Step 1). The size of the model is determined by the extent to which the study area is partitioned and how many time periods are being analyzed (Step 2). This is a three-commodity problem (Step 3). The sources of each commodity are inputs to the right-hand side supply vectors, $S^{(k)}$ (Step 4). The controls being considered are represented as alternative paths or control links. In this illustration, control alternatives exist for commodity 2 (Step 5). The description of transport processes are embodied in the individual commodity continuity matrices $A^{(k)}$ and the intercommodity interdependency matrices, $D^{(k',k)}$ (Step 6). The control costs are summarized in the coefficients $C^{(2T)}$ of the objective function and the technological limits on controls are expressed by the control bounds (Step 7). The performance standards are specified in terms of commodity 3 (Step 8). No specific institutional constraints are included in this illustration (Step 9). Lastly, the problem is solved using a selected computational algorithm (Step 10). The post-optimal analysis of the solution is included in this last component.

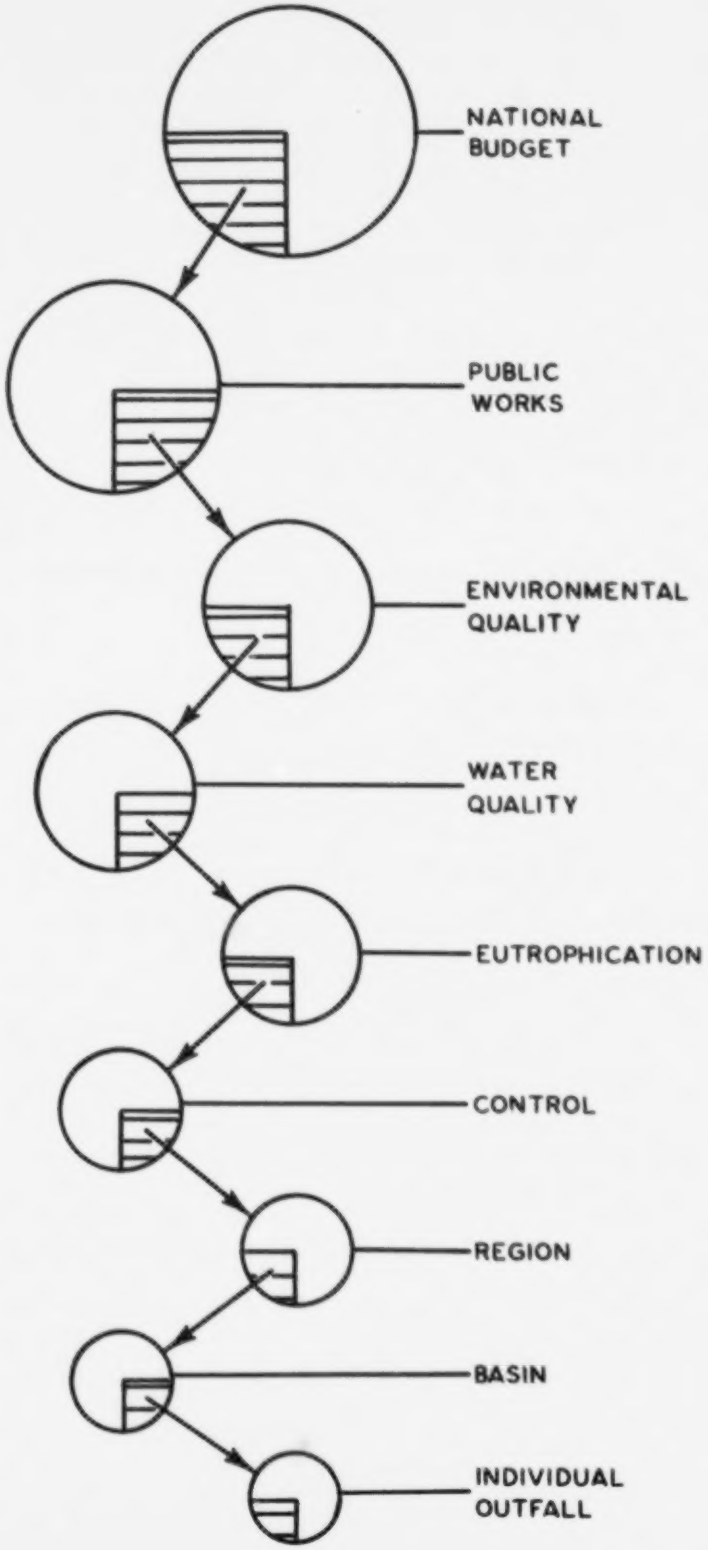
$$\left.\text{MINIMIZE } Z = \left[C^{(2T)}\right]' \left[Q^{(2T)}\right] : \text{ CONTROL COSTS}\right\} \text{OBJECTIVE FUNCTION}$$

SUBJECT TO:

$$
\left.
\begin{array}{l}
\begin{array}{l}
\text{LINKS}
\end{array}
\\
\left[\begin{array}{l|c|c|c|c}
 & \text{WATER} & \text{COMMODITY } \textcircled{2} & \text{COMMODITY } \textcircled{3} & \text{CONTROL} \\
\hline
\text{WATER NODES} & A^{(1)} \text{ CONTINUITY} & 0 & 0 & 0 \\
\hline
\text{COMMODITY } \textcircled{2} \text{ NODES} & D^{(1,2)} \text{ INTERDEP.} & A^{(2)} & 0 & T^{(2)} \\
\hline
\text{COMMODITY } \textcircled{3} \text{ NODES} & D^{(1,3)} \text{ INTERDEP.} & D^{(2,3)} \text{ INTERDEP.} & A^{(3)} \text{ CONTINUITY} & 0 \\
\hline
\text{COMMODITY } \textcircled{3} \text{ STANDARDS} & P^{(1,3)} \text{ COUPLING} & P^{(2,3)} \text{ COUPLING} & P^{(3,3)} \text{ COUPLING} & 0 \\
\hline
\text{CONTROL BOUNDS} & 0 & 0 & 0 & I
\end{array}\right]
\end{array}
\left[\begin{array}{c}
Q^{(1)} \\ \text{WATER} \\ \text{VECTOR} \\
\hline
Q^{(2)} \\ \text{COMMODITY} \\ \textcircled{2} \\ \text{VECTOR} \\
\hline
Q^{(3)} \\ \text{COMMODITY} \\ \textcircled{3} \\ \text{VECTOR} \\
\hline
Q^{(2T)} \\ \text{CONTROL} \\ \text{VECTOR}
\end{array}\right]
=
\left[\begin{array}{c}
S^{(1)} \\ \text{WATER} \\ \text{SUPPLY} \\
\hline
S^{(2)} \\ \text{COMMODITY} \\ \textcircled{2} \\ \text{SUPPLY} \\
\hline
S^{(3)} \\ \text{COMMODITY} \\ \textcircled{3} \\ \text{SUPPLY} \\
\hline
\underline{Y}^{(3)} \\ \text{COMMODITY} \\ \textcircled{3} \\ \text{STANDARD} \\
\hline
\overline{Q}^{(2T)} \\ \text{COMMODITY} \\ \textcircled{2} \\ \text{CONTROL} \\ \text{UPPER} \\ \text{BOUNDS}
\end{array}\right]
\right\} \text{CONSTRAINTS}
$$

$$\left[Q^{(1)}\right], \left[Q^{(2)}\right], \left[Q^{(3)}\right], \left[Q^{(2T)}\right] \geq 0$$

FIGURE 4. ILLUSTRATIVE MATHEMATICAL PROGRAMMING FORMULATION OF PROBLEM


NATIONAL BUDGET
PUBLIC WORKS
ENVIRONMENTAL QUALITY
WATER QUALITY
EUTROPHICATION
CONTROL
REGION
BASIN
INDIVIDUAL OUTFALL

SUMMARY

It is important to emphasize that the management model, like other models, seeks to assist us in analyzing complex systems. Its purpose is to provide information regarding the problem at hand. It does not, and cannot provide all of the information which is needed in the decision making process.

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DISCUSSION

MANAGEMENT MODELS

FRUH: One technical point, which came up yesterday morning in the second session and also goes back to Don O'Conner's model. We can take measurements and get productivity and nitrogen and phosphorus and find out how many algae in terms of chlorophyll for mass. However, the biggest difficulty, technically, now, is to take that algal mass and say, this is the problem in the algal mass - the odor. Then this is the dollar sign for removing the odor in the water treatment plant. Or, this is the algal bloom, and this is the mat that floats into the shoreline and this is the dollar value that has gone down on the land.

BREZONIK: Some of that has to be done on a stochastic basis. There are things you cannot predict.

FRUH: But that's what we checked in our research.

BREZONIK: You just don't know, for example, when the wind is going to blow from the south, and blow your algal blooms into the shore, where your setting may be. Based on past records you can say that 80% of the time the wind does blow from the south, and perhaps make some sort of prediction based on that, or how much of a problem you are going to have with the scum layer on shore based on productivity in the lake.

PATTEN: Can I ask if you have any information whether the technology is available to solve a multiple objective problem simultaneously?

HEANEY: You can solve them to the extent that you have multiple goals.

PATTEN: Do you have to weight these and lump them into some sort of an index, and then maximize and minimize that index?

DYSART: We are using some words here and I think we ought to clarify them. I would say that we must distinguish between objectives and goals. I look upon goals as being extremes. Goals - such as water quality standards. I think that an objective could be either the minimization, as stated here, of cost, subject to certain environmental quality, or maximizing quality, subject to your budgetary constraints. Your comments regarding whether it has to be an extreme situation - I think that an optimal level of development of use in a river basin, whatever the optimum is, is going to be a blend of different purposes and uses of the water resources.

PATTEN: It is a rather involved type of thing, and it is never extreme in any one sense.

DYSART: I think we have to talk about a blend of purposes, in that case there will be this trade-off. There are a number of purposes of development. Qualitative oriented purposes, such as recreation or aesthetics. We have quantitative oriented purposes such as flood control and power, and some of these are complementary, some of them are very competitive. We would probably be in trouble if we had to say, formulate what blend these were going to be.

HEANEY: I would just like to make one short point: I think we have become accustomed, after a day and half, to thinking about modeling, in general, as a dynamic process, we are not looking for a one shot deal as far as typical modeling processes. We get an optimal solution. We have a simulated model, we have all of the inputs. Based on that we go back, re-do some of our homework, and go ahead with the same learning process. I just don't want to get static.

SCHAAKE: I would like to ask the panel to comment on how they see interaction between this kind of a framework for doing this problem process and the world of the rest of society, the world of political process, and whether or not we are dealing with a problem that might be called a sociological problem, in a technological sense, or a technological problem in a sociological sense. I think there is a difference between objectives and constraints.

HEANEY: Dr. Shapiro suggested an eutrophication index, and we had a real lively discussion. Everybody is sitting here and they've got their own definitions, and he was boldly suggesting a single index. Now, the dollar, in my opinion, is this index. Not the best. Perhaps there is a better one. I would like to have an alternative that allows us at least to measure the relative value.

REYNOLDS: Basically, John had a good point, that the dollar may not be a sacred measure of all this. There are deficiencies. It probably is the best one we've got to work with, but we do need to recognize some of the deficiencies. Address yourself to prices, for instance, we have the prices of commodities that are reflected by the market index. We can find out the price of a particular agricultural commodity, or particular stocks and bonds, and things like this. The market may not always work perfectly. If the market worked perfectly, prices should be the reflection of people's tastes and preferences. We have other indications, and that is political process. Maybe part of this comes through in our system in the United States. We temper the prices. It could happen with the benefit-cost ratio. Benefit-cost ratios in the past have been, well, there are devaluating procedures now, but they would measure the tangible benefits and they would recognize the intangible benefits, so that if they went past that point where there was maximum benefit, you see they were bringing in part of the intangible benefit within a non-market commodity, so there are sometimes other things in our system besides prices that have a significance.

GIBBS: I would like to say, John, that pertaining to modeling, if you or anybody else would give us some insight into how we can measure the eutrophic model.

SCHAAKE: The way to make progress is to look at those assumptions (we're back on the road block) and use the model to gain insight into what's happening.

HEANEY: That's exactly what Eckstine's models, the economist's models, modeled, and they have been very successful. They were awarded the Nobel prize in modeling. Their models were extremely simple.

SHELDON: We have the uncertainty of risk, and for many of us, particularly in eutrophication problems, pesticide contamination, and so on, and that is the uncertainty and the possibility of some way to reverse that. I think perhaps the thing you are trying to analyze in this case would be to minimize risk. I know there are ways to do this. This is certainly a criterion that doesn't always come to the fore.

GIBBS: I would like to comment on the assumption of the independence of utility companies to lower production costs. The business of the economic work that has been done in water pollution control has recognized the fact. They term this the relationship between production function, and consumption function, or vice versa, as analogous of the pulp mill producing waste in the stream and its effect on the fisherman downstream. So there is a relationship between the production function of the pulp mill and the consumption function of the fisherman downstream. Conceptually they have recognized this and tried to get measurements.

SCHAAKE: I also see the possibility of this whole program running in more than one way. I don't see the uniqueness of the solution. I live where Thoreau lived and I love it there. I think the environment is great. I wouldn't want to live anywhere else. I just believe that we could have more of that. This is part of the value of the lake.

MOREAU: We are talking about possible objectives, to translate all of this to equivalent market situations. What I suggest is that when we come back to the basic decision, about the eutrophication process, we come to the question of environmental quality, the same questions relating to redistribution of income. The environmental quality is becoming an object unto itself the same as redistribution of income, and is not something to be counted in the benefit-cost calculations. The system is something separate.

SCHWARTZ: I think the political decision has already been made, and we don't have to calculate the values. It's just like, people decide if we're going to have ballet, or we're going to have museums, we're going to have education, we're going to have trips to the moon, or we're going to have national security. We don't have to calculate the benefits of this. The decisions have been made. I think what happened recently, or what is happening now in Congress relates to this. In the President's budget there was 200 million dollars for water quality program, facilities, and so on. The House of Representatives upped this at one time to a billion dollars. It got back down to 600 million. The Senate now has gone to a billion and there is going to be a compromise and it may be six hundred million. I think this sort of exercise you are doing here is a problem in how to achieve these objectives at minimum cost, it is not how to go through benefit-costs and justify it.

GIBBS: Are these adequate funds to clean every lake in the United States?

SCHWARTZ: We are developing our technology. There may be cheaper ways of doing this. The way we are doing it now may not be the best way.

GIBBS: Some allocation has to be made. Why would we clean this lake and not this lake?

SCHWARTZ: I think that in terms of supply and demand, which we are dealing with, there is a multiple dimensional demand function that exists.

MOREAU: We don't reach those decisions in the market place. The point is, we reach it through a political process.

PATTEN: I would like to know how many private dollars are invested in this meeting, for example. I regard ourselves as externalities of the major public interest. This is public costs. The whole operation of all public service is represented. Most of us are on the public payroll in one way or another.

MOREAU: That is just the overhead.

PATTEN: This is a pretty critical operation. I don't think that industry has done a comparable operation. What is being done about it? My understanding is that these problems are solvable right now, technically. In relation to the power structure.

BARTSCH: I am not sure I understand your question. Is your question really asking this: Is industry doing something with relation to eutrophication?

PATTEN: I know they are doing something, but I don't know what they are doing.

BARTSCH: We are aware that there are a considerable number of industries that are involved very deeply in the problem of improving the environment. One industrial complex, which is the soap and detergent industry, is vitally concerned with the problem of eutrophication, for obvious reasons, the inclusions of phosphorus in detergent formulations. This industry is involved in a national Government-industry joint effort to explore many of the aspects of eutrophication. I am not sure I can state the number of dollars that are being contributed to various activities.

EUTROPHICATION MANAGEMENT MODELING

Summary Statement

by

The Panel Members

EUTROPHICATION MANAGEMENT MODELING

Following an opening statement by the Chairman, the discussion was opened for comments from the participants. A summary of these comments is contained below.

Opening discussion indicated the lack of adequate models for predicting the response of the lake to control of nitrogen and phosphorus. Thus, at least in the near future, it will be necessary to make control decisions without a good understanding of the expected changes in water quality and the benefits to be derived.

The problem of multiple objectives was discussed briefly. It was pointed out that the optimal level of development will be a blend of different purposes and uses of the water resources. Computationally, the multiple objective problem can be transformed into an equivalent single objective function with other goals included as constraints. (Charnes _et_. _al_., 1967).

Animated discussions followed regarding the appropriateness (or even relevance) of attempting to measure the benefits of effectiveness of environmental control. It was suggested that environmental quality is an object unto itself, the same as redistribution of income, and is not something to be counted in the benefit-cost calculus. The use of a monetary measure of the "value" of environmental control also was challenged because of its inadequacies.

The panel agreed that the dollar is certainly an incomplete measure of value. However, it is, at present, the best available index and allows us to measure the relative desirability of alternatives. If the market worked perfectly, prices should reflect people's tastes and preferences. We have other indicators through the political process which may alter the prices. Benefit-cost ratios in the past have measured tangible benefits, and recognized the intangible benefits. Thus, if they were bringing in the intangible benefits of a non-market commodity, they could deviate from the point where net measured benefits were maximized.

One of the participants felt that the political decisions with regard to the environment have been made and we don't need to calculate values. He felt it was analogous to a public decision for museums, education, and national security. Given the proposed national commitment to environmental control, he felt it was unnecessary to go through benefit-cost analysis to justify it.

A panelist responded by asking whether these funds would be adequate to clean every lake in the United States. He did not think so and emphasized that even with large sums of money, it will be necessary to establish priorities and make allocations. Environmental control must compete with other uses since the supply of money is far less than the demand. One of the panelists has prepared a chart showing how this allocation process works from the national level on downward. This chart is shown on the next page.

Final discussions dealt with the extent to which the private sector has made commitments to environmental control. It was pointed out that industry has been involved in solving this problem for a number of years. However, their commitment has been unimpressive in comparison with the size of their profits.

In summary, eutrophication management modeling is in an embryonic stage of development. There is little information available on the alternative control measures and the response of a lake to these controls. Consequently, eutrophication control decisions are, and will continue to be, made in a highly speculative framework.

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SUMMARY SESSION

DR. BARTSCH: I just want to say "Thank you," and I am sure I do this for all of us, for the excellent job that Hugh Putnam and his associates have done in arranging this meeting here in St. Petersburg.

Next, I would like to say that I have been listening, giving some thought, and have discussed with a limited number of people how we might continue this good start I feel we have made. I came here knowing nothing about modeling and I am now, hopefully, I believe, at least a little less a dummy than I was.

We have covered a lot of ground; there was an excellent give and take. No one here was timid or reticent as to what ought to be said. I sense a current of feeling that we ought to continue this effort, we shouldn't lose what we have started here. As I said yesterday, there are a number of obvious mechanisms by which we can keep this thread of communication alive. Let me offer a possibility to consider. I have the impression that it would be advantageous now to do two things.

The first would be to project ourselves now to more specific examples of the application of modeling and use this as the main one of perhaps two themes of a future meeting. One thing that might be worth considering is to take some specific projects which are now susceptible to modeling, put them through modeling, bring them up at a subsequent meeting, and examine them. It would then be possible to see where the deficiencies are and where the values are. If this seems reasonable, I would like to point out that in our own national program we have two such projects that would be susceptible to this kind of treatment. One of these concerns a small lake, Shagawa Lake in northern Minnesota on the Canadian border. It has only substantial tributary inflow and three small ones. It is a flow-through lake. Here is located the city of Ely, which is one of those peculiar towns that has a population of four thousand people in the winter time when it is thirty below zero, and twelve thousand in the summer time, because it is the jumping off spot into the Boundary Waters Canoe Area. This lake is eutrophic, in whatever definition you wish to use today. For some sixty years or more this city has accounted for some 10-80 percent of the phosphors input. We have conducted algal assays here to assess the significance of the city's input. From the work we have done, it is considered worthwhile now to go to a full scale advanced water treatment plant to remove nutrients. We expect the plant to be in operation by roughly a year from this coming spring. We expect to remove 95 percent or more of the phosphorus from this sewage, which we are confident we can do, and this will reduce the total input by 70 or 80 percent. This offers very suitable modeling circumstances, I would think.

Another lake that could be considered has a number of sources of nutrient input. This is Sallie Lake located near Detroit Lakes, Minnesota. It is a site at which we are planning to intensively harvest waterweeds during the summer of 1970. Included in the background information are data on the nutrients, nutrient budgets, productivity patterns, hydrology including surface inflow and outflow, ground water, and related information. Table top scrutiny indicates that we ought to be able to remove substantial nitrogen, phosphorus, iron, and other nutrient elements in relation to the annual rate of input.

Here is another possibility. I am sure from the work that you people have talked about we could find material about other lakes that might be the theme for such a meeting.

The second element of a subsequent session was suggested by Dr. Patten which is to broaden the scope and consider how we might pursue the aspects that involve people and the too infrequent use of the pill, the problems of population growth, and the resulting impact on the environment.

I would like to hear any comments you may wish to express.

DR. LEE: I strongly support your approach. I would like your comment on how you would get the data out to the people ahead of time to give us something to work on so that we come to the meeting better prepared.

DR. BARTSCH: What we talked about among ourselves was that with respect to these two cases that I dealt with specifically, we don't have the capability of modeling in our laboratory, but we need to get together a small group of consultants and put this together maybe six or eight months ahead of the meeting then it could be sent out so that everybody will have time to take a look at what it is and what we can do with it and what the deficiencies are.

DR. PATTEN: I was going to suggest, if it hasn't been suggested, I don't know how many people you anticipate, probably the same size as this?

DR. BARTSCH: I would think so. I think that this has been an exceedingly manageable sized group.

DR. PATTEN: I would suggest that if it is at all feasible you set it up in some place where you have access to computing facilities, and remote terminals and other things of this type, so that you can make it, in fact, a workshop.

DR. BARTSCH: My colleagues and I have considered this. I also considered offering perhaps the best site in the United States for such a meeting, which is Corvallis, Oregon. We in our laboratory have a computer terminal and other resources at Oregon State University.

DR. SANDERS: I would hope that very soon we could get together on these compartments. I think that getting a group together on a model such as this at this time that you might be a little premature. We need to get the people that are interested and involved to sit down and find out what we need to get involved with.

DR. PATTEN: I agree with that, but again, you want to define the objectives of such a workshop, if you in fact want to produce a valid model of a system. That is one thing. But, if you want to produce an exercise where you can draw in the people of the various backgrounds and let them interact through the medium of the exercise, that would be another objective, and I suggest that would be the more valid objective.

DR. SANDERS: Unless you have the right inputs into the system you are not going to get what you want out of it.

DR. PATTEN: What you want out of it, though, is the inter-action experience of working together. If you want some real answers out of a real system, that's another objective.

DR. BELLA: If you want to get some feedback to the data, the model results feedback to the data, back to the model.

DR. PATTEN: I would like to call your attention to an experience I had just a few weeks ago. At a conference on environment and pollution, there were young people involved, and it was suggested to them that they, themselves, try to resolve the pollution problem there at the conference. One of the inputs into the model was to clear the room of cigarette smoke, and you would be amazed. At first the answer was that they could not solve this problem. The simile here is that I think it would be possible to throw people of diverse backgrounds together and let them thrash out what the difficulty really is of working this thing out.

DR. BARTSCH: I think in a sense this could be done. Let me suggest that if you have some thoughts that you haven't yet expressed with respect to how we might continue from this point, and especially in relation to the meeting, or meetings, please write to me about it. For myself, I would like to say thank you to each of you for being here. You are not only being helpful to yourselves, but to the nation, and specifically to us in this type of experimental work.